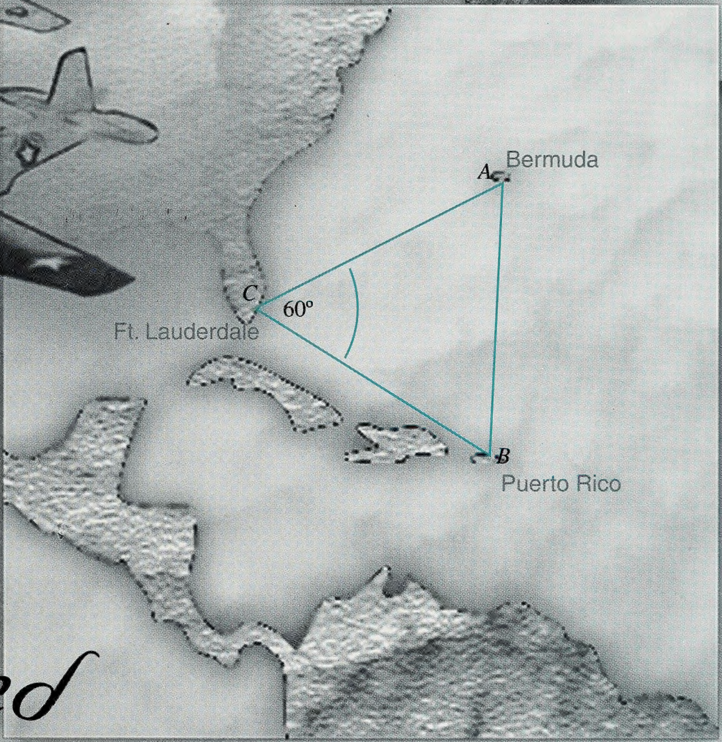
Module 3



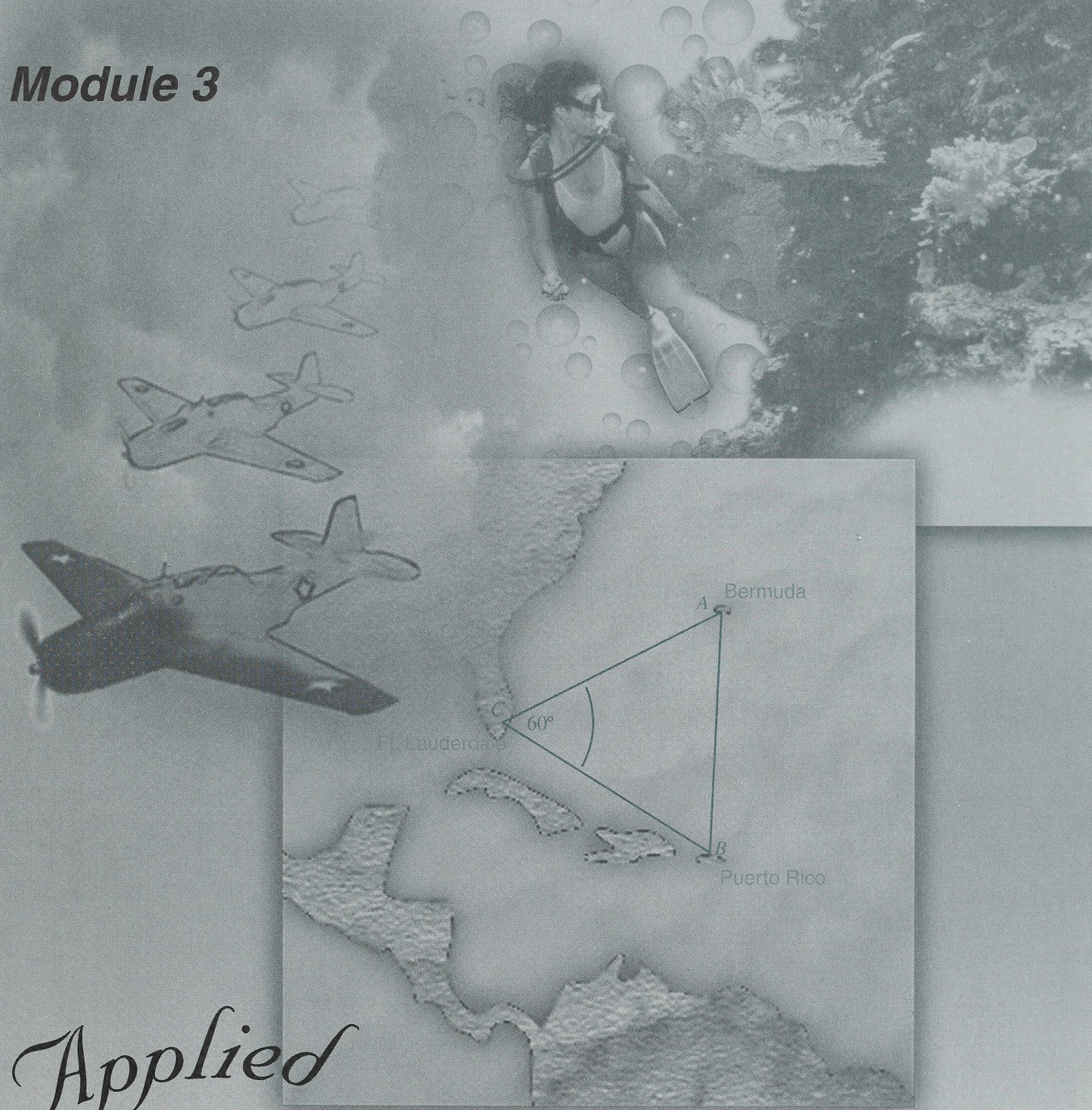
Applied Mathematics 10

RELATIONS AND FUNCTIONS

Learning Technologies Branch

Alberta
LEARNING

Module 3



Applied Mathematics 10

RELATIONS AND FUNCTIONS

Learning Technologies Branch

Alberta LEARNING

Applied Mathematics 10
Student Module Booklet
Module 3
Relations and Functions
Learning Technologies Branch
ISBN 0-7741-1722-2

Cover photo: COREL CORPORATION

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	

The Learning Technologies Branch has an Internet site that you may find useful. The address is as follows:

http://www.learning.gov.ab.ca/ltb

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

Welcome

Applied Mathematics 10

Module 1
Measurement

Module 2
Number Patterns in Tables

Module 3
Relations and Functions

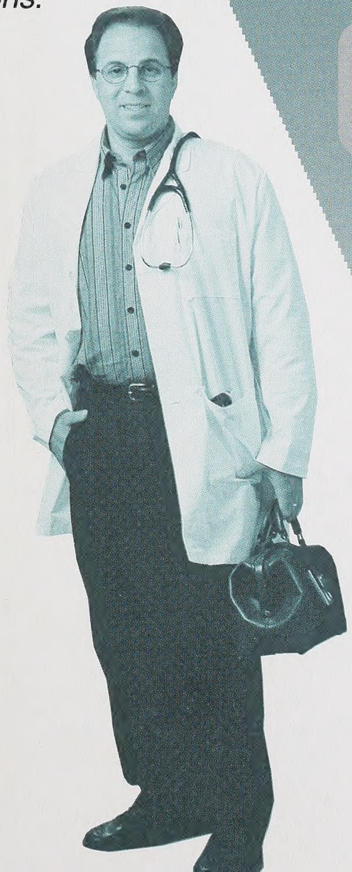
Module 4
Sampling

Module 5
Line Segments

Module 6
Linear Functions

Module 7
Trigonometry

Welcome to Module 3. We hope you'll enjoy your study of Relations and Functions.



Applied Mathematics 10 contains seven modules and a final test. Work through the modules in the order given, since several concepts build on each other as you progress through the course.

Contents

Module Project: Epidemics

Submit the Module Project.

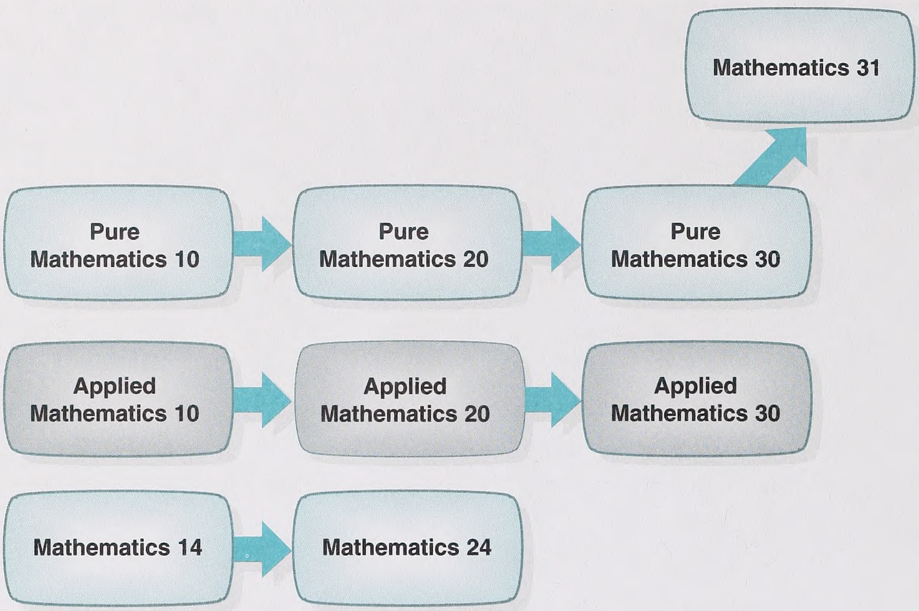
Module Summary

Submit the Module Assignment.

Appendix

Introduction to Applied Mathematics 10

Applied Mathematics 10 is the first course in the Applied Mathematics 10–20–30 program of studies. Another program of studies is Pure Mathematics 10–20–30; students who complete Pure Mathematics 30 often choose to take Mathematics 31. A third program of studies is Mathematics 14–24.



Each mathematics program is designed for students with different mathematical strengths and interests.

- Pure Mathematics 10–20–30 is intended for students who are strong in algebra and mathematical theory.
- Applied Mathematics 10–20–30 is better suited to students who prefer to solve problems using numerical reasoning or geometry.
- Mathematics 14–24 is a general mathematics program for high school students who have experienced difficulties in previous mathematics courses.

Each sequence of courses is designed for students with different post-secondary and career plans.

You may find it helpful to read mathematics updates on Alberta Learning's website:

http://www.learning.gov.ab.ca/studentprograms

Before enrolling in Applied Mathematics 10, it is recommended that you talk with a school counsellor about your career plans.

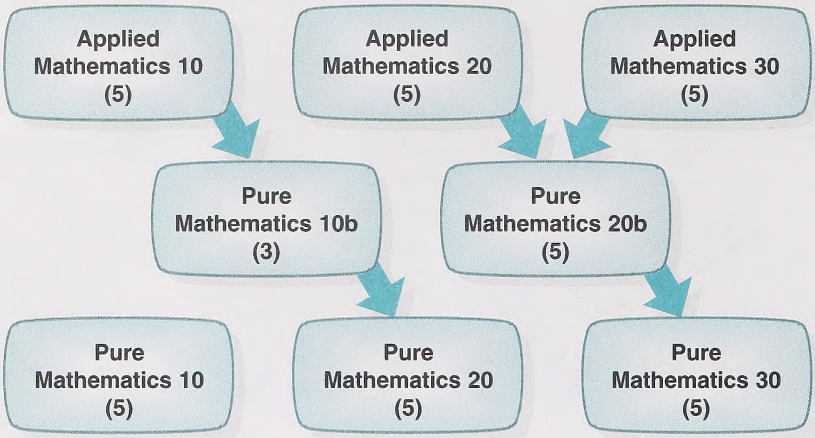
TRANSFERRING FROM THE APPLIED MATHEMATICS PROGRAM

You should be aware that the applied and pure mathematics courses do have some topics in common; other topics are specific to either the applied or pure mathematics courses.

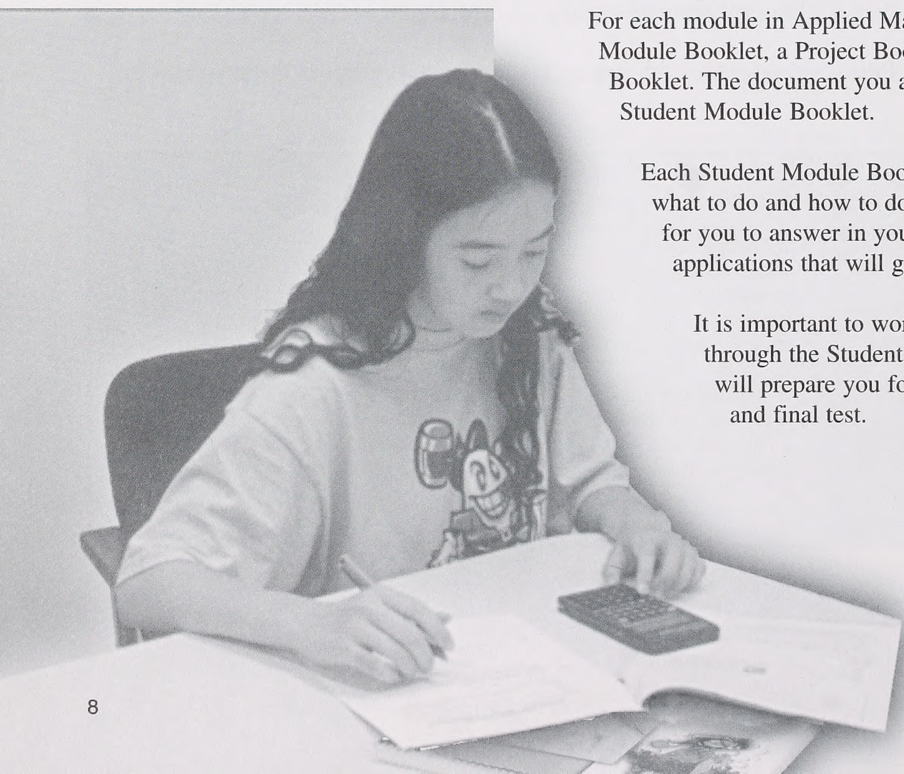
The following table shows some of the common topics and some of the independent topics in the mathematics program.

Applied Topics	Common Topics	Pure Topics
• linear programming • data tables and trends • design and layout • metric and imperial measure • data presentation • vectors and matrices • periodic, fractal, and recursive patterns • financial decision making • costing and design problems	• spreadsheets • line segments and linear graphs • scaling • triangles • surveys • financial mathematics • quadratic functions • circle geometry • the bell curve	• irrational numbers • exponents • polynomial and rational expressions • mathematical expectation • growth patterns • linear and non-linear systems • operations on functions • mathematical reasoning • exponential and logarithmic functions • conics • combinations • trigonometric functions

If you want to transfer from the Applied Mathematics 10–20–30 sequence to the Pure Mathematics 10–20–30 sequence at a future time, you won't have to repeat the topics that are common to pure mathematics and applied mathematics. If you decide to transfer to Pure Mathematics 20 after successfully completing Applied Mathematics 10, you may take the 3-credit course called Pure Mathematics 10b. If you decide to transfer to Pure Mathematics 30 after successfully completing Applied Mathematics 20 or Applied Mathematics 30, you may take the 5-credit course called Pure Mathematics 20b. The two bridging courses are shown in the following diagram.



STRATEGIES FOR COMPLETING APPLIED MATHEMATICS 10



For each module in Applied Mathematics 10, there is a Student Module Booklet, a Project Booklet, and an Assignment Booklet. The document you are presently reading is called a Student Module Booklet.

Each Student Module Booklet will show you, step by step, what to do and how to do it. There are readings, questions for you to answer in your mathematics binder, and applications that will give you hands-on experience.

It is important to work systematically and carefully through the Student Module Booklets. This work will prepare you for the projects, assignments, and final test.

Following are some suggestions for organizing your mathematics binder:

- Keep a section of your binder to record your responses to the questions in the Student Module Booklet. Also, store your marked assignments here.

- Keep a section of your binder for work in progress on your projects. Keep your research notes, plans, rough drafts, and so on.

- Keep a section of your binder to record new skills and concepts as well as important results and formulas. Get in the habit of describing new skills and concepts in your own words, and recording useful ways to help you remember what a concept means. Make charts and diagrams to help you connect mathematical ideas.

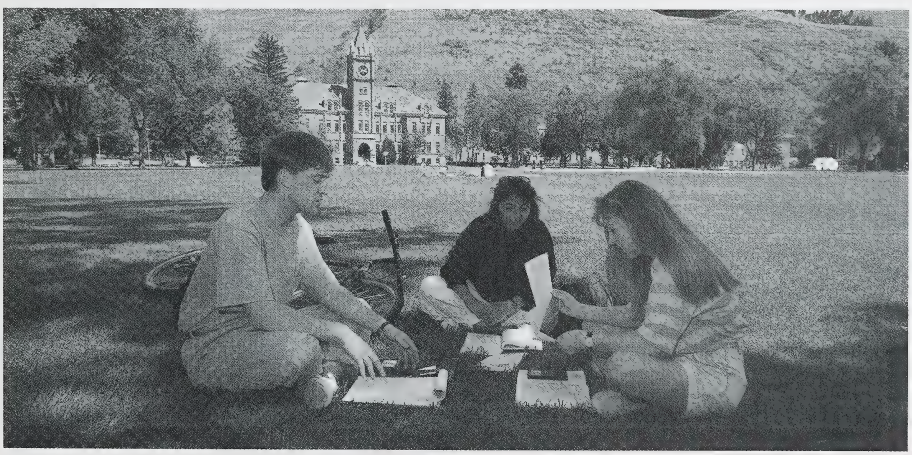
- Keep a section of your binder to record mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you grasped a difficult concept (an "aha!" experience) or when you used a calculator or spreadsheet in a new way.

Mathematical Processes

Throughout this course, you will be expected to perform the following mathematical processes:

- Connect mathematical ideas to everyday experiences and to concepts in other disciplines.
- Develop and use problem-solving strategies.
- Reason and justify your answers.
- Communicate mathematical ideas.
- Select and use appropriate technologies to solve problems.
- Develop and use estimation and mental-math strategies.
- Use visualization to assist in processing information, making connections, and solving problems.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Applied Mathematics 10 to be your study partner. You will find that having a friend with whom you can discuss mathematical ideas will make your studying more enjoyable.



Resources You Will Need

In addition to the distance learning materials for Applied Mathematics 10, you will need the following resources:

- the *Addison-Wesley Applied Mathematics 10 Source Book,* Western Canadian Edition, published by Addison Wesley Longman Ltd. (1999)

- a binder, lined loose-leaf paper, graph paper, dividers, a pencil, and an eraser

- metric and imperial measuring devices, such as a ruler, a tape measure, a yardstick, a vernier caliper, and a micrometer

- a mathematical instrument set (compass, protractor, and triangles)

- a computer installed with a spreadsheet program

  Note: Two popular spreadsheet programs are *ClarisWorks*™ and *Microsoft® Excel*. The examples in this course show *Microsoft® Excel.*

- a graphing calculator

  Note: Where it is applicable, the examples in this course and the textbook show the TI-83 calculator; however, all of the graphing calculators in the following chart are approved for use on tests.

Brands	Texas Instruments	Sharp	Casio
Models	TI-82 TI-83 TI-83 Plus TI-86 TI-89 TI-92 TI-92 Plus	EL-9600c EL-9600* EL-9300* EL-9200*	CFX-9850Ga-Plus CFX-9850G* CFX-9800G* FX-9700 series*

*no longer commercially available, but may be available on loan from your school division

If you intend to use the TI-83 or TI-83 Plus graphing calculator, it is recommended that you view the video *The TI-83 Graphing Calculator Video Tutor* to discover some of the calculator's features.

Many of the resources you will need for this course may be purchased from the Learning Resources Distributing Centre (LRDC). Following is the LRDC website:

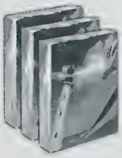
http://www.lrdc.edc.gov.ab.ca

You may wish to discuss the availability of resources with your teacher, as your school division may have a loan policy.

Visual Cues

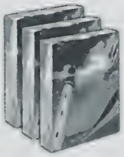
You will find many visual cues in this course. Colour is used to highlight terms that are defined in the Glossary of the Appendix of each Student Module Booklet. You will also find several icons in the margins. Read the explanations given to discover what the various icons prompt you to do.

- Refer to the textbook.



- Use the companion CD for Applied Mathematics 10.

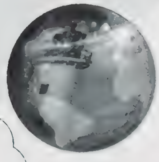
- Use mathematical instruments, measuring devices, and other materials.



- Work with a computer.

- Complete the module project or assignment.

- Explore the Internet.



Remember: Any Internet website address given in this module is subject to change.

MODULE OVERVIEW

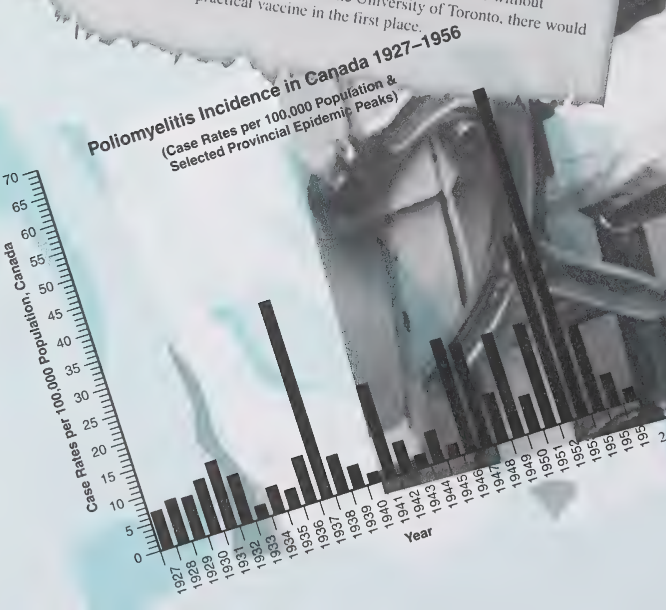
AVENTIS PASTEUR LIMITED

"40 YEARS OF POLIO PREVENTION!

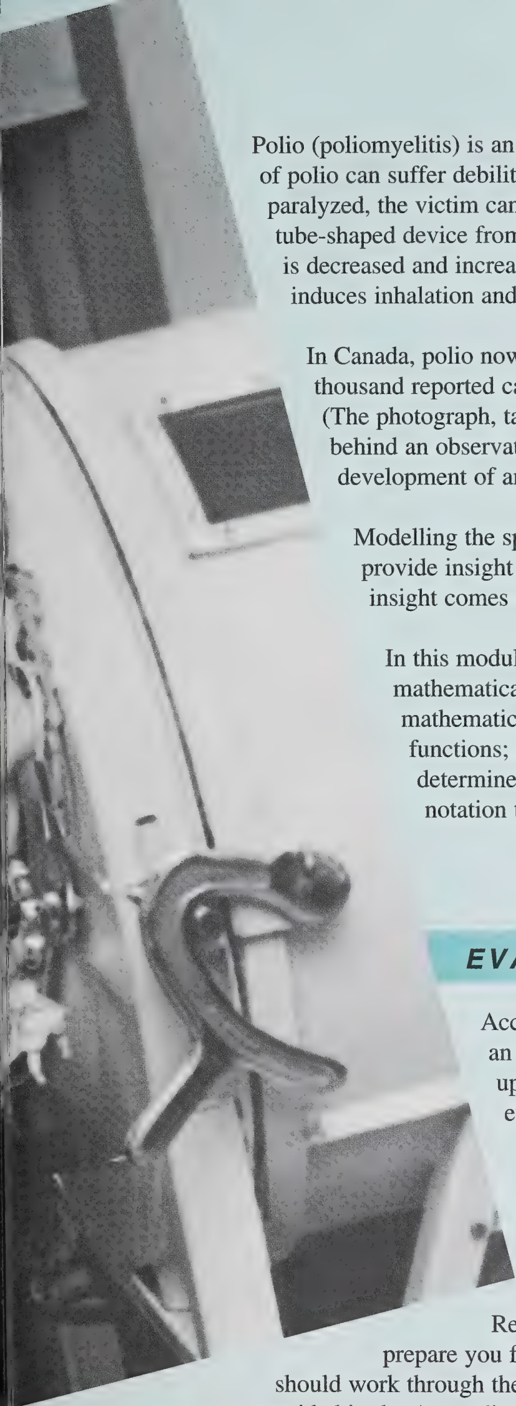
CANADA AND THE GREAT SALK VACCINE TRIAL OF 1954-55"

By Christopher J. Rutty

Forty years ago the largest medical experiment in history took place to test a vaccine to prevent the escalating ravages of poliomyelitis. This was the Salk vaccine named after Dr. Jonas Salk of Pittsburgh. Close to two million children across the United States and parts of Canada were involved in this field trial, which was orchestrated by the National Foundation for Infantile Paralysis (NFIP), or March of Dimes. Canadian involvement in this massive experiment went far beyond testing a small amount of vaccine. The Canadian role was fundamental to the entire project, for without Connaught Medical Research Laboratories at the University of Toronto, there would not have been a trial, or a practical vaccine in the first place.



CORBIS/BETTMANN



Polio (poliomyelitis) is an infectious disease that attacks the central nervous system. A victim of polio can suffer debilitating paralysis. When the chest muscles and diaphragm are paralyzed, the victim can suffocate. To enable breathing, a patient is placed in an iron lung, a tube-shaped device from which only the head protrudes. The air pressure within the container is decreased and increased rhythmically to move the chest walls. The changing air pressure induces inhalation and exhalation and breathing occurs without the patient expending effort.

In Canada, polio now rarely occurs; however, from 1927 to 1962, there were almost 50 thousand reported cases. The disease came in epidemics involving thousands of cases. (The photograph, taken in 1952, shows anxious parents visiting their sick children from behind an observation window.) Polio epidemics were finally halted with the development of an effective vaccine and a widespread vaccination program.

Modelling the spread of an epidemic with a mathematical relation is one way to provide insight into the biological nature of the disease transmission. With this insight comes understanding and, hopefully, effective control of the disease.

In this module, you will study mathematical relations and a special class of mathematical relations, called functions. You will represent data using mathematical relations; plot data using appropriate scales; draw the graph of functions; describe functions in terms of ordered pairs, a rule, and a graph; determine the domain and range of a relation from its graph; and use function notation to represent and evaluate functions.

EVALUATION

Accompanying this Student Module Booklet is a Project Booklet and an Assignment Booklet. Your grading in this module will be based upon the module project and the module assignment you submit for evaluation. The mark distribution is as follows:

Module Project	**40 marks**
Module Assignment	**60 marks**
TOTAL	**100 marks**

Remember that Activities 1 to 5 in this Student Module Booklet will prepare you for completing the module project and the module assignment. You should work through these activities carefully and compare your answers with the suggested answers provided in the Appendix.

The Follow-up Activities provide extra help and enrichment. You may choose to do some or all the questions in the Follow-up Activities. Again, you should compare your answers with the suggested answers provided in the Appendix.

[1] Christopler J. Rutty, "40 Years of Polio Prevention!" 1995, <http://www.healthheritageresearch.com/PolioHistory.html> (30 March 2000). Reprinted by permission.
[2] Ibid.

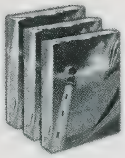
MODULE PROJECT
EPIDEMICS

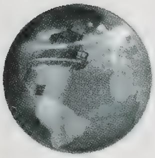
Beginning the Project

An epidemic is a widespread occurrence of a disease. The disease may spread across a community, a continent, or the entire world. Epidemics have claimed victims on a scale associated with the carnage of war. Fortunately, much has been learned about diseases and how they spread. This knowledge has reduced, but not eliminated, the devastation brought about by many epidemics.

Your module project for Module 3, Relations and Functions, is Epidemics. In this project you will study how epidemics spread. You will prepare a general description of an epidemic and give reasons why the spread of an epidemic eventually slows and finally comes to a stop. You will report on two epidemics of your choice.

Your project will be marked on the completeness and accuracy of the information and on the mathematical representation of the patterns and numbers related to the spread of epidemics.





To begin your project, turn to page 110 of the textbook, read the page, and complete the questions posed. Store the responses in the project section of your mathematics binder.

Use the Internet, magazines, newspapers, encyclopedia, and other books to research epidemics. You may find it useful to begin your research by visiting Addison Wesley Longman Ltd.'s Internet site, described on page 111 of the textbook. You may also wish to visit the following Internet site:

http://www.discovery.com/exp/epidemic/epidemic.html

As well, you may go to the following site and do a search for epidemics:

http://www.britannica.com

In this project, you will focus on the numbers associated with the spread of epidemics. In order to describe and analyse these numbers, you will need a good understanding of mathematical patterns and relations. As you work through Activities 1 to 5, you will develop your ability to describe patterns and relations.

As you work through the activities, continue your research for this project. You will be given more direction on how to complete this project later in this module. In the meantime, you may want to talk about this project with others. Just remember that the work you submit must be your own.

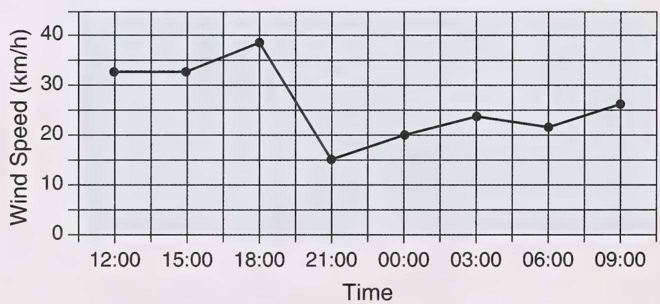
ACTIVITY 1

PAUL A. SOUDERS/CORBIS

Interpreting Graphs

Pincher Creek, in southern Alberta, has the reputation of being one of the windiest places in the province.

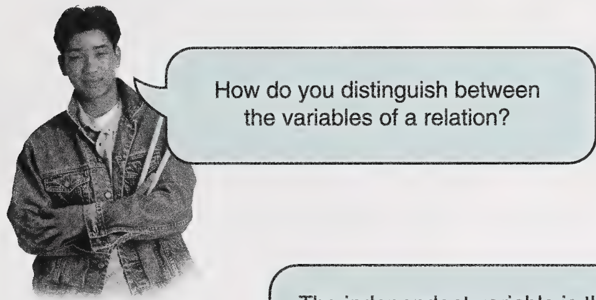
The following graph shows wind-speed measurements at Pincher Creek on September 22, 1999; the measurements were made at 3-h intervals from midday (noon).

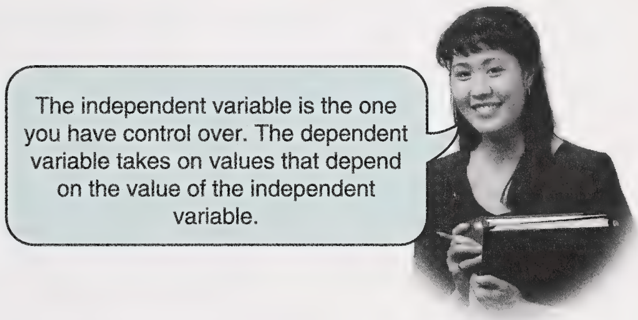


You can see that on this particular day in Pincher Creek the wind speed was initially steady. Then, the wind speed increased slightly. In the hours after 18:00, the wind speed dropped to its lowest recorded level. From then, the wind speed increased gradually until 03:00, then decreased, and then became stronger again.

Notice that the points in the graph are joined by line segments.

For the relation shown on the preceding graph, time is the **independent variable** and speed is the **dependent variable**. Generally, when graphing, the independent variable of a relation is on the horizontal axis and the dependent variable is on the vertical axis.

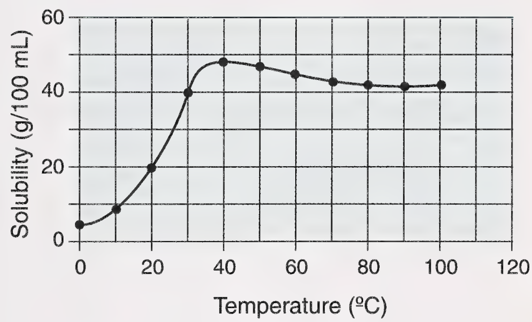




The researcher decided the time intervals at which the wind speed in Pincher Creek would be measured, so the time periods are the independent variables. The wind speed is the dependent variable, since the speed measured depends on the time the measurement is taken.

Consider another example. The amount of sugar or salt that you can dissolve in water increases as the temperature of the water increases. On the other hand, sodium sulphate (used in the production of detergent and in the dyeing of textiles) does not follow this pattern. The amount that you can dissolve in water at various temperatures varies as shown in the given graph.

Solubility Curve for Sodium Sulphate

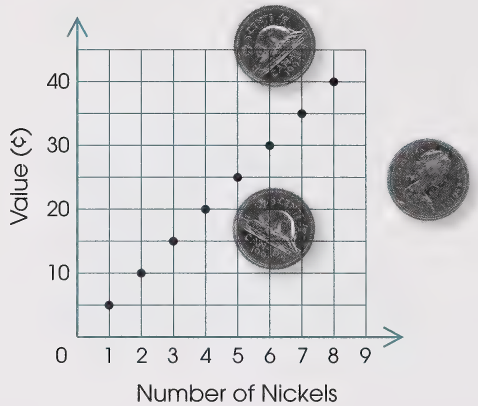


Notice that the data points in the graph have been connected by a smooth curve (instead of line segments) to produce the solubility curve.

1. Refer to the graph of the solubility curve for sodium sulphate.

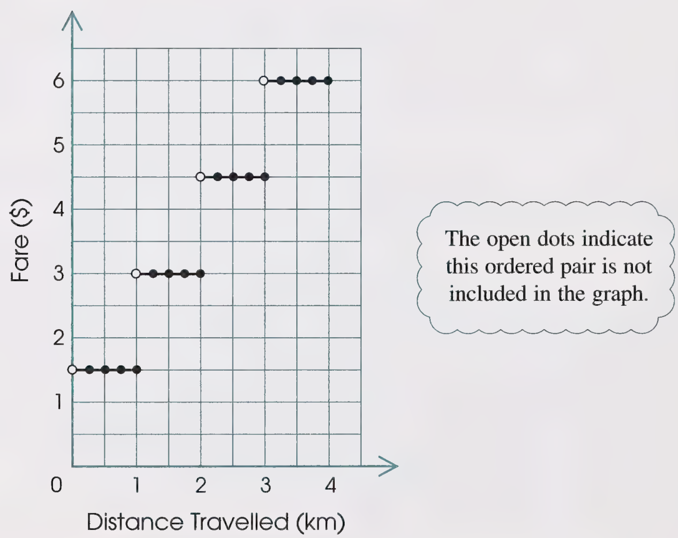
a. What is the maximum solubility for sodium sulphate?
b. At what temperature does the maximum solubility occur?
c. Explain why it is appropriate to connect the data points.

2. The following graph shows the relationship between the number of nickels and the value of the nickels in cents.



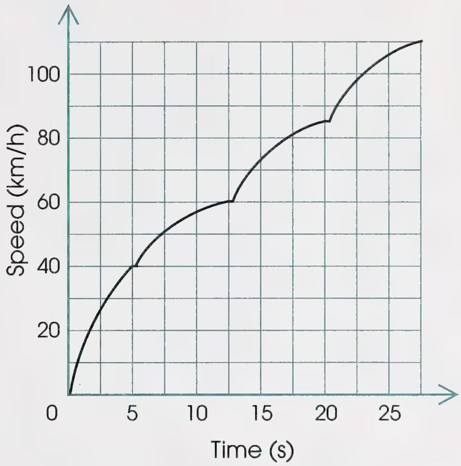
a. What is the value in cents of 2 nickels? 5 nickels? 8 nickels?
b. Explain why it is not appropriate to connect the data points.

3. A taxi driver charges a fare of $1.50 for each kilometre or part of a kilometre travelled.



a. What is the minimum fare charged?
b. What is the fare charged for travelling 0.5 km? 1 km? 1.25 km? 2 km?
c. Explain why it is appropriate to have open dots on this graph.

4. A car has a 4-speed manual transmission. The following graph shows how the speed of this car increases to highway speed.

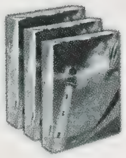


a. At what speeds does the driver change gears? Explain your answer.
b. Explain why it is appropriate to connect the data points.

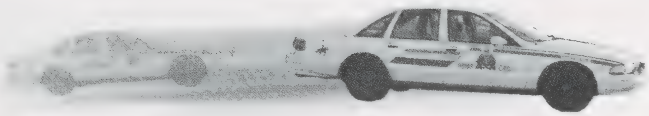
Compare your responses with the suggested answers in the Appendix, Activity 1, pages 60–61.

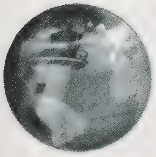
Usually graphs have scales on the axes, but this is not essential. Even without the scales, a graph can effectively indicate how the dependent variable of a relation changes with respect to the independent variable.

The direction of the curve as you go from left to right indicates whether the dependent variable increases, remains constant, or decreases. The highest and lowest parts of the graph correspond to the maximum and minimum values of the dependent variable.



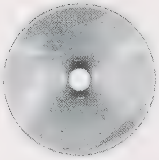
Turn to pages 112 and 113 of the textbook and read the opening three paragraphs of Tutorial 3.1, "Interpreting and Creating Graphs." Then work through "Example 1: Creating a scenario to match a graph." As you work through this example, think about a possible explanation for the change in the height of the water in the bathtub. Write down your thoughts and then compare your explanation with the scenario given.





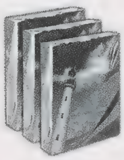
How would the graph showing the scenario of a car passing another vehicle look? To discover how this situation can be represented by a graph, visit the following website:

http://hal.grasslands.ab.ca/kinematics/fs.html



To discover the relationship between time and height (above the ground) for a person riding on a Ferris wheel, view the segment "Ferris Wheel" on the companion CD.

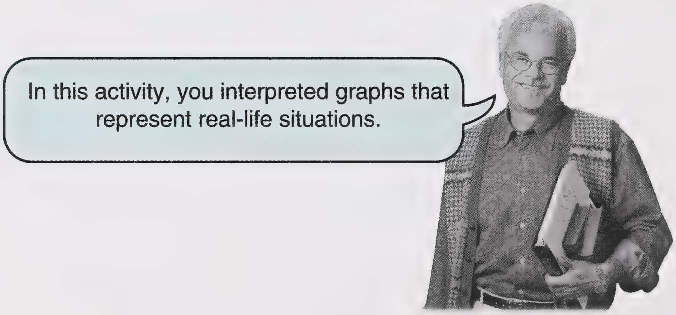
To discover how ball bounces can be represented by a graph, view "Ball Bounce 1" and "Ball Bounce 2" on the companion CD.



5. Turn to pages 115 and 116 of the textbook and answer exercises 2, 3, and 4 of "Exercises: Checking Your Skills."

Compare your responses with the suggested answers in the Appendix, Activity 1, page 61.

LOOKING BACK



6. In the journal section of your mathematics binder, explain how you decide when to connect the points in the graph of a relation.

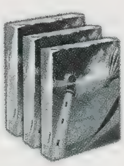
Compare your response with the suggested answer in the Appendix, Activity 1, page 61.

ACTIVITY 2

Graphing from Tables of Data

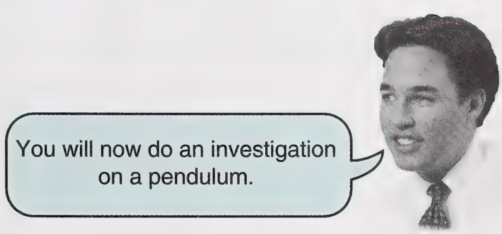
When you go on vacation you probably take a camera with you or you buy postcards. That is because a picture better illustrates what you have seen and experienced than your words can.

In mathematics, a graph of a mathematical relation is a picture of the relation. The graph often highlights patterns more powerfully than a rule or a table.



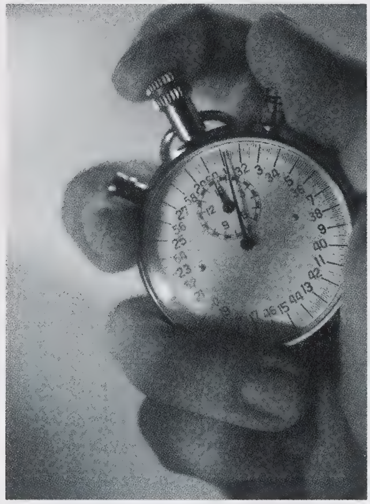
1. Turn to page 117 of the textbook and (using paper and pencil) make a scatterplot of the data in Tutorial 3.2, "Graphing from Tables of Data: Practise Your Prior Skills." **Note:** You will need graph paper. Let height be the independent variable.

Compare your response with the suggested answer in the Appendix, Activity 2, page 62.

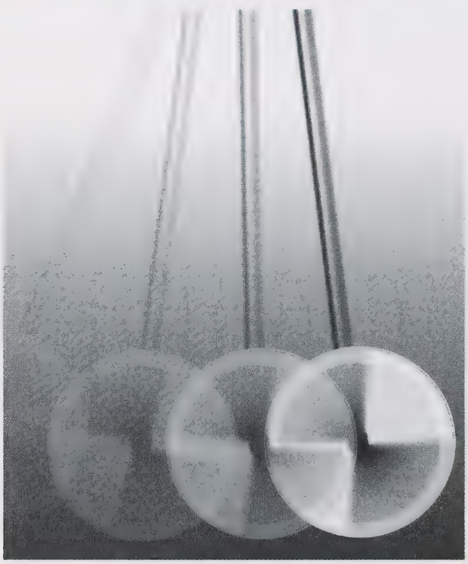


2. Turn to pages 117 and 118 of the textbook and read the introductory paragraphs of "Investigation 1: Length and Frequency of a Pendulum." Then answer exercises 1 to 6 of the investigation.

   If possible, find someone to help you perform the investigation. You will need a stopwatch or a watch with a second hand.



Compare your responses with the suggested answers in the Appendix, Activity 2, pages 62–64.



You have discovered that the duration of time for a particular pendulum to make each swing is constant. If the length of the pendulum can be adjusted, the pendulum can be made to swing faster or slower.

Thus, a pendulum clock that has been gaining or losing a few minutes each day can be made to keep time accurately by adjusting the pendulum length.

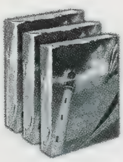
If you would like to learn more about how pendulum clocks work, visit the following website:

http://www.howstuffworks.com/clock.htm

PLOTTING DATA FROM TABLES

Before you actually begin plotting data, examine your graphing calculator. Notice that the keys are colour-coded to help you locate the keys you need more easily. For example, on the TI-83 graphing calculator, the grey keys are number keys. The blue keys on the right side of the calculator are operation keys. (Notice that the subtraction key is blue, while the negative key is grey.) The blue keys across the top are graphing keys. The black keys are editing keys, scientific calculator keys, and advanced-function keys.

The primary function of each key is printed in white. The secondary function is printed in yellow above the key. To access the secondary function of a key, you must first press the yellow $\boxed{\text{2nd}}$ key. For example, to access π (pi), press $\boxed{\text{2nd}}$ $\boxed{\wedge}$. In this course, you will see this key sequence written as $\boxed{\text{2nd}}$ [π].



Turn to page 400 in the textbook and work through "Utility 6: Resetting the TI-83 Default Settings." This utility explains how to reset the default (or factory) settings on the TI-83 calculator.

Now, turn to pages 401 and 402 of your textbook and work through exercises 1 to 5 of "Utility 7: Using the TI-83 to Plot Data from Tables." **Note:** In exercise 2, you will need the following data on the percent of women in the Saskatchewan legislature.

Year	Percent of Women in Saskatchewan's Provincial Legislature
1975	3.3
1980	1.6
1984	7.8
1988	7.8
1993	18.2

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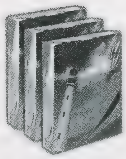
Now work through the following example to discover some shortcuts when using a graphing calculator to plot data from tables.

Example



Miki likes to bowl. The following table shows Miki's scores for 6 games.

Game	1	2	3	4	5	6
Score	88	96	84	108	90	110

Use a graphing calculator to display the scatterplot of this data.

Solution

Step 1: Use Method 1 or Method 2 to enter the given data.

Method 1: Press (STAT) (1) **or** (STAT) (ENTER) and clear any previously entered data. To do this, use the procedure described in exercise 1 of Utility 7 on page 401 of your textbook.

Enter the given data in columns L1 and L2. Use the procedure described in exercise 2 of Utility 7 on page 401 of your textbook.

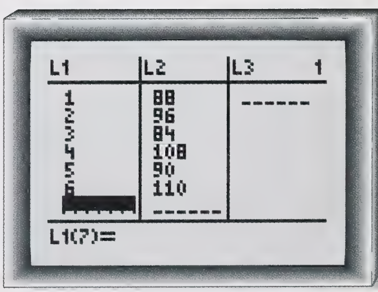
Method 2: Press the following key sequence:

(STAT) (ENTER) (2nd) [QUIT]

(2nd) [{] (1) (,) (2) (,) (3) (,) (4) (,) (5) (,) (6)
(2nd) [}] (STO➧) (2nd) [L1] (ENTER)

(2nd) [{] (8) (8) (,) (9) (6) (,) (8) (4) (,) (1) (0)
(8) (,) (9) (0) (,) (1) (1) (0) (2nd) [}] (STO➧) (2nd)
[L2] (ENTER)

Press (STAT) (1) **or** (STAT) (ENTER) to confirm that the data has been entered correctly.



Step 2: Set your calculator to graph the data. Press Y= and clear any previously entered equations. Press 2nd [STAT PLOT] ENTER and ensure the following selections are made.

To change settings, use the arrow keys and move the cursor to the desired word or icon and then press ENTER.

Step 3: Press ZOOM.

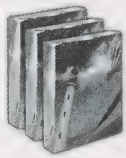
The screen shown at the right will appear.

Notice that there is an arrow after the 7 at the bottom of the screen. This means that there are more choices. Use the arrow keys to move the other choices into view. There are 10 choices altogether.

Press 9 ENTER to select "**9: ZoomStat**."

The graph at the right appears in the screen.

Pressing ZOOM 9 automatically changes the WINDOW settings so the graph of the data will fit on the screen.



3. Return to page 117 of the textbook and the table in "Practise Your Prior Skills."

   Use your graphing calculator to display the scatterplot of this data. **Note:** Assume that height is the independent variable and shoe size is the dependent variable.

4. Pat and Kelly discussed how a hot beverage cools over a period of time. They theorized that the rate at which the temperature of the beverage decreases is greatest just after it is poured. Once the beverage is lukewarm, the cooling continues at a slower rate.

   In order to test their theory about cooling rate, Pat used a thermometer to measure the temperature of hot water right after it was poured from a kettle into a cup. Leaving the thermometer in the water, Pat read the temperature every 5 s and Kelly recorded the readings.

   Kelly recorded the following temperatures.

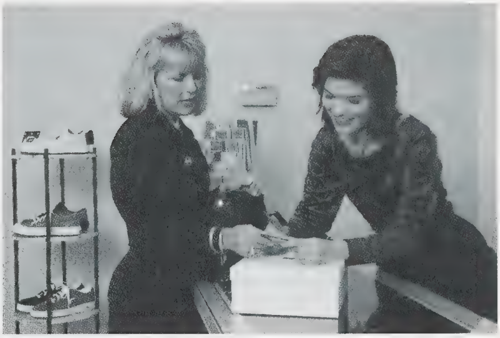
Time (s)	Temperature (°C)
0	98
5	90
10	83
15	76
20	70
25	65
30	60
35	56
40	52
45	49

Time (s)	Temperature (°C)
50	46
55	43
60	41
65	39
70	37
75	35
80	33
85	32
90	31

   a. Use your graphing calculator to make a scatterplot of the data.
   b. Use the graph to determine whether the girls' theory was right.

Compare your responses with the suggested answers in the Appendix, Activity 2, page 65.

5. The size of women's shoes in Europe differs from the sizes used in North America. In general, the European shoe size for women is 30 more than the comparable size (rounded up) in North America.



a. The given chart shows how European sizes are related to North American sizes.

Make a paper-and-pencil scatterplot of the data.

Note: Put the North American sizes on the x-axis and the European sizes on the y-axis.

Women's Shoe Sizes

North American Sizes	European Sizes
6	36
6.5	37
7	37
7.5	38
8	38
8.5	39
9	39

b. The given chart shows how North American sizes are related to European sizes.

Make a paper-and-pencil scatterplot of the data.

Note: Put the European sizes on the x-axis and the North American sizes on the y-axis.

Women's Shoe Sizes

European Sizes	North American Sizes
36	6
37	6.5 and 7
38	7.5 and 8
39	9

Compare your responses with the suggested answers in the Appendix, Activity 2, page 66.

When you make a scatterplot from a table of data, you match each independent value of the relation with its dependent value(s) to obtain a set of ordered pairs. Then you graph these ordered pairs.

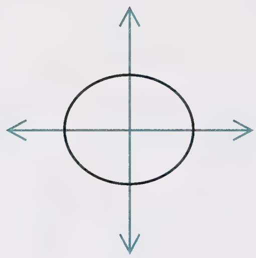
If there is one and only one dependent value (y-value) for each independent value (x-value), the relation is a **function**. If there is more than one dependent value (y-value) for an independent value (x-value), the relation is not a function.

There is a vertical-line test for a function. If two points on the graph of a relation can be joined by a vertical line, the relation is **not** a function. If no two points can be joined by a vertical line, the relation is a function.

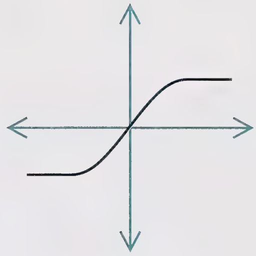
Example

Is each of the following a function?

a.



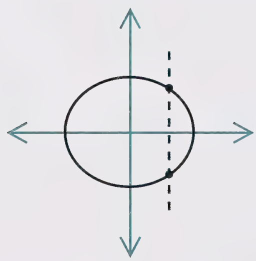
b.



Solution

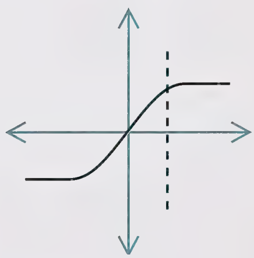
Use the vertical-line test to determine if each graph is a function.

a.



Since there is a vertical line that intersects the graph more than once, the relation is **not** a function.

b.

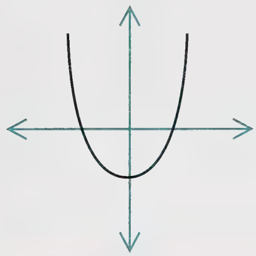


Since there is no vertical line that intersects the graph more than once, the relation is a function.

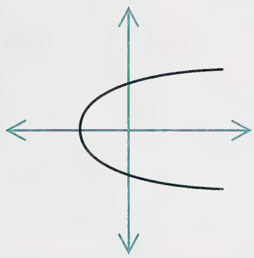
6. Examine the relations in questions 5.a. and 5.b. of this activity. Are these relations functions? Why or why not?

7. Examine the following graphs. State whether or not each is a function. Justify your answers.

a.

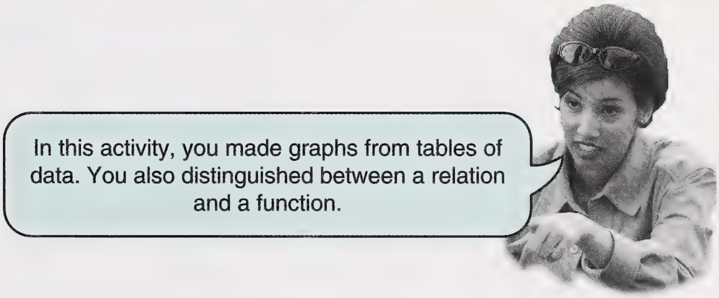


b.

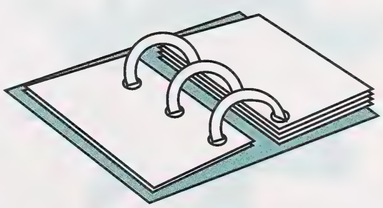


Compare your responses with the suggested answers in the Appendix, Activity 2, page 66.

LOOKING BACK



8. In the journal section of your binder, describe what is meant by a relation. Then explain when a relation is a function. Give examples and non-examples.



Compare your response with the suggested answer in the Appendix, Activity 2, page 67.

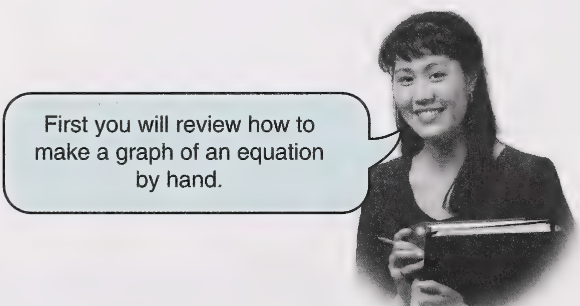
ACTIVITY 3

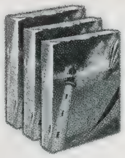
Graphing from Equations

Have you ever noticed that at the top of a tall waterfall, the water falls as a fairly solid stream, and by the time it nears the bottom, the water is reduced to a thin, wispy spray? One reason for the break up of the stream is the interaction of the water and the air; but the main reason is the way things fall due to gravity. The force of gravity stretches any segment of a stream vertically—the longer it falls, the more it stretches.

The formula $d = 5t^2$ is a compact way of describing the relationship between the distance (d) a water globule drops and the time (t) that has elapsed since the water globule fell over the edge of the waterfall. You have probably encountered many other formulas in school and your daily life.

In this activity, you will graph equations and formulas to discover more about the functions they represent. You will make these graphs using a paper-and-pencil method and using a graphing calculator.





1. Turn to page 123 of the textbook and examine Method 1 of "Example 1: Graphing Equations in x and y."

   Show the calculations that were required to make each of the tables in Example 1. **Note:** As a sample, the calculations for the first row of each table are shown here.

Example 1.a)

If $x=-2$, then

$$\begin{aligned} y&=2x-1\\ &=2(-2)-1\\ &=-4-1\\ &=-5 \end{aligned}$$

Example 1.b)

If $x=-1$, then

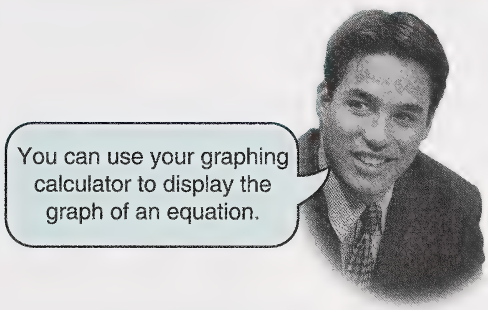
$$\begin{aligned} y&=2^{x}\\ &=2^{-1}\\ &=\frac{1}{2} \text{ or } 0.5 \end{aligned}$$

2. Graph the following functions using pencil and paper. **Note:** You will need graph paper.

   a. $y=5x-3$

   b. $y=2x^{2}+3x-1$

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 67–68.



Take out your graphing calculator and work through the following example.

Example

Use technology to graph the equation $y=5x-2$.

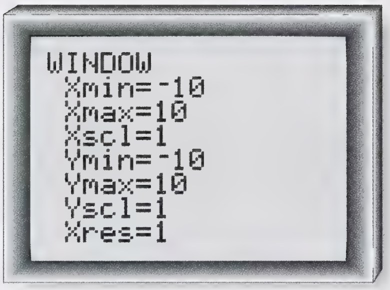
Solution

Step 1: Because a graphing calculator can be used to graph scatterplots as well as equations, check the STAT PLOT menu. Press 2nd [STAT PLOT]. (In other words, press 2nd Y=.)

A screen like the following is displayed.

Ensure all stat plots are turned off.

Step 2: Press WINDOW to check the WINDOW menu. Use the default settings shown for this equation.



A quick way to reset the WINDOW menu to the default settings is to press ZOOM 6.

Step 3: Press Y=. A screen like the following is displayed.

Use the arrow keys and press CLEAR to remove any equation to the right of an equal sign.

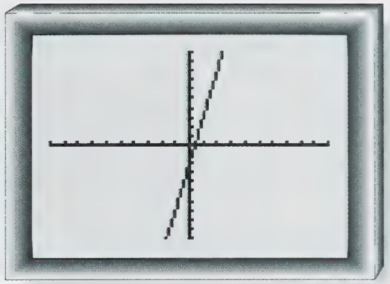
To enter the required equation $y = 5x - 2$, press the following key sequence.

Be sure to press the blue minus key, not the grey negative key.

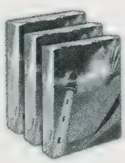
5 X,T,Θ,n – 2 ENTER

This key enters the variable x.

Step 4: To graph the equation, press GRAPH. The following graph appears.



You can use the graphing calculator to display the table of values that corresponds to a graph display. Moreover, it is possible to change this table display.



Turn to page 405 of the textbook and work through "Utility 9: Working with Tables: Viewing a Table of Values."

3. Turn to page 127 of the textbook and answer exercises 1.c., 1.d., 1.e., and 1.h. of "Exercises: Checking Your Skills." **Hint:** Graph each equation on your graphing calculator. Use the corresponding table of values to find the value of y when $x = 0$.

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 69–70.

4. Turn to the Appendix in this booklet and examine the graphs of each of the equations in question 3. (That is, examine the graphs for textbook exercises 1.c., 1.d., 1.e., and 1.h. of "Exercises: Checking Your Skills," page 127.)

   a. What are the equations whose graphs are straight lines? How are these equations similar?

   b. What are the equations whose graphs are curves? How are these equations similar?

c. Explain why all the graphs represent functions.

Compare your responses with the suggested answers in the Appendix, Activity 3, page 71.

So far, you have only used the default WINDOW settings. Work through the following example to discover how to change the default WINDOW settings to get a better view of a graph.

Example

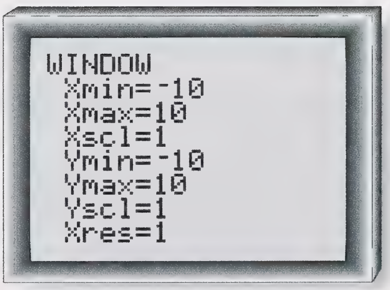
Graph the equation $y = 20 - 5x^2$.

Solution

Step 1: Press 2nd [STAT PLOT] to check the STAT PLOT menu.

Ensure all stat plots are turned off.

Step 2: Press WINDOW to check the WINDOW menu. Use the default WINDOW settings.



A quick way to reset the WINDOW menu to the default settings is to press ZOOM 6.

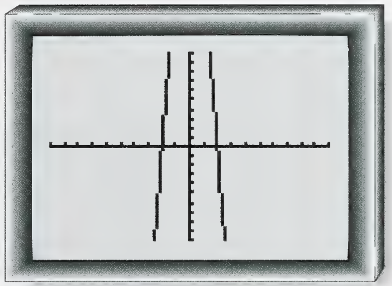
Step 3: Press [Y=], clear any expressions that appear on the screen, and then press the following key sequence to enter the equation $y = 20 - 5x^2$.

[2] [0] [−] [5] [X,T,Θ,n] [x^2]

Be sure to use the blue minus sign, not the grey negative sign.

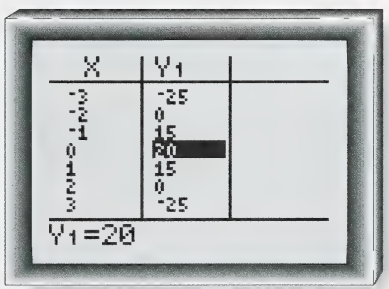
Press [GRAPH].

The following graph will be displayed.



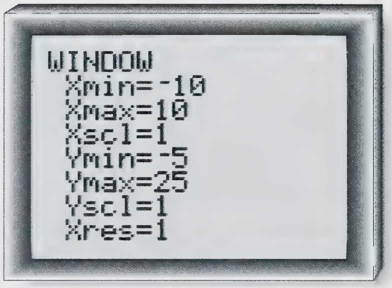
Notice that the top of the curve is cut off in this display. The reason for the top being cut off is that the default settings show all the y-values between -10 and 10, but the highest y-value for this graph is greater than 10.

Step 4: Estimate the highest value of y on the graph. One way to do this is to access the table of values. Press [2nd] [TABLE] and then use the arrow keys to find the value of y when $x = 0$.



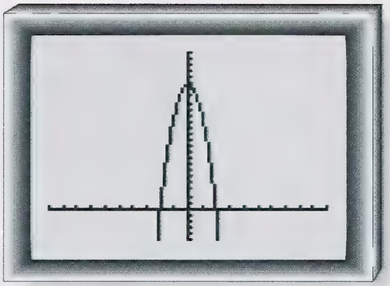
The highest y-value is about 20.

Step 5: Press WINDOW to access the WINDOW menu. This time use the following WINDOW settings.



Be sure to use grey negative keys, not the blue subtraction key, for -10 and -5.

Step: 6 Press GRAPH. The following graph is displayed.

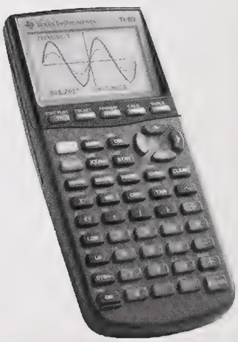


Notice that a clearer view of the curve is obtained with these WINDOW settings.

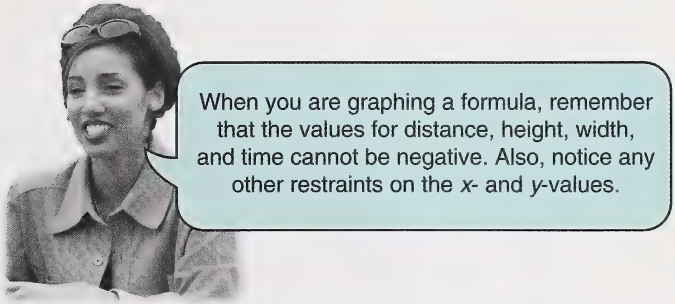
5. a. Use the default WINDOW settings to graph $y = x^2 - 15.5$.

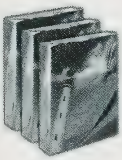
b. Use the TABLE feature to estimate the lowest value of y.

c. Adjust the WINDOW settings to get a clearer view of the curve.



Compare your responses with the suggested answers in the Appendix, Activity 3, pages 71–72.





Take out your calculator, turn to page 125 of the textbook and work through "Example 2: Graphing and interpreting formulas." **Note:** Stop at the bottom of page 125. Do not clear the calculator.

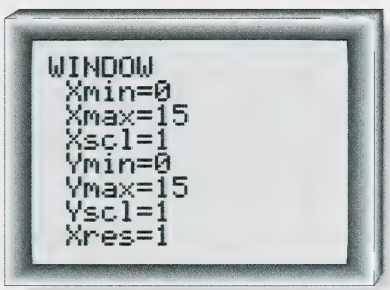
6. Answer the following questions about the example.

a. Why was a minimum x-value of 0 chosen?

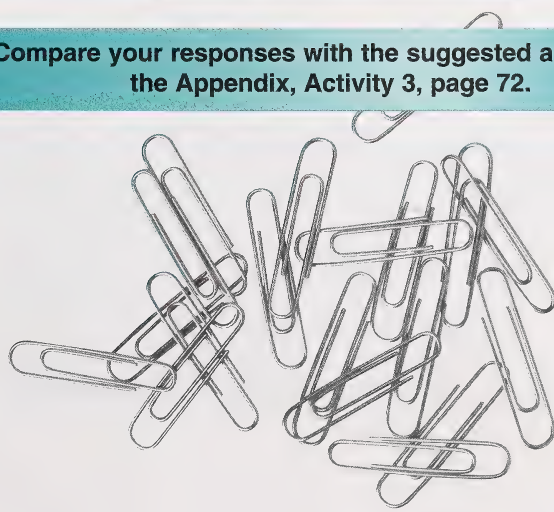
b. Why was a minimum y-value of 0 chosen?

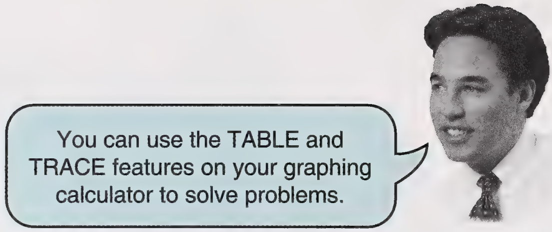
c. Press 2nd [TABLE] to access the table view. Estimate the highest value of y.

d. Will the following WINDOW settings give a clear view of the curve? Explain.



Compare your responses with the suggested answers in the Appendix, Activity 3, page 72.





Work through the following example.

Example

The height (h) in centimetres of an adult female is related to the length (r) in centimetres of the person's radius (one of the forearm bones).

The relationship is described by the following formula:

$$h = 3.34r + 81.2$$

Estimate the height of a woman whose radius is 20 cm long.

Solution

Step 1: Rewrite the formula using x- and y-variables.

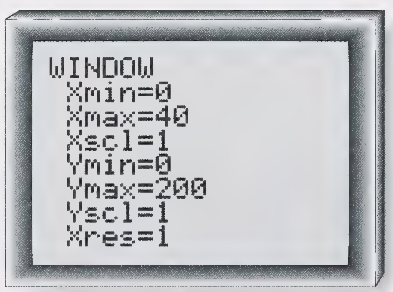
$$y = 3.34x + 81.2$$

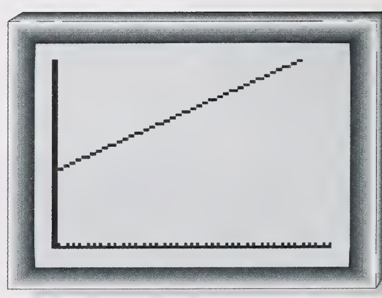
Step 2: Determine the restrictions on the x- and y-variables. Because the length of the radius cannot be negative, the minimum x-value is 0. Because height cannot be negative, the minimum y-value is 0.

Step 3: Press 2nd [STAT PLOT] to check the STAT PLOT menu. Ensure all stat plots are turned off.

Step 4: Select appropriate WINDOW settings, keeping in mind the restrictions on the x- and y-values, and graph the equation $y = 3.34x + 81.2$.

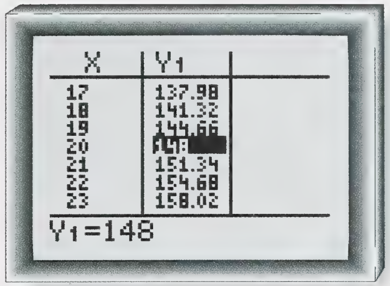
Following is a sample.





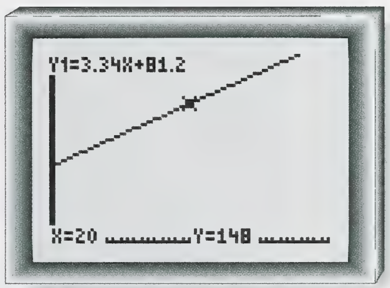
Step 5: To solve the problem, find the value of h when r is 20.

One way to do this is to access the TABLE view.

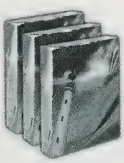


The woman's height is about 148 cm.

Alternatively, you can use the TRACE feature. Press TRACE and notice the flashing cursor (point) on the graph. Also notice the x- and y-values at the bottom of the graph. Use the left and right arrow keys to find the value of y when $x = 20$.



The woman's height is about 148 cm.



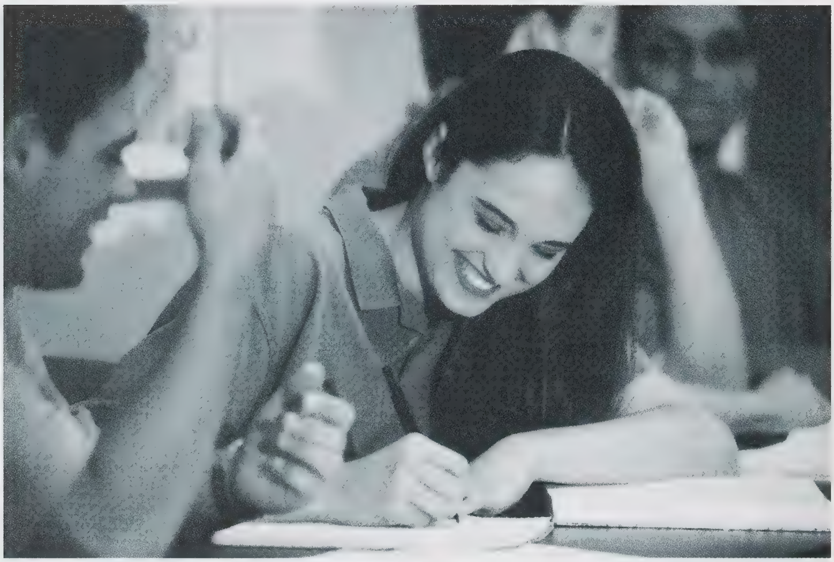
7. Turn to pages 127 and 128 of the textbook and answer exercises 2 and 4 of "Exercises: Checking Your Skills."

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 73–75.

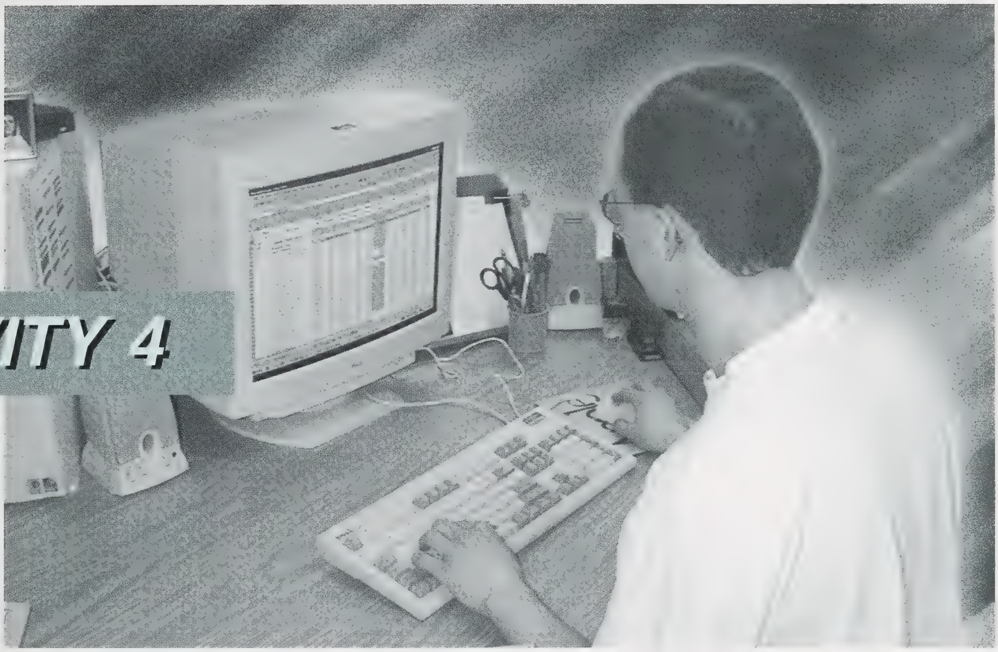
LOOKING BACK

In this activity, you made graphs from equations and formulas to discover more about the functions they represent. You made these graphs using a paper-and-pencil method and using a graphing calculator.

8. In the journal section of your notebook, explain some of the advantages of using paper and pencil to graph a function. Also, explain some of the advantages of using a graphing calculator.



Compare your response with the suggested answer in the Appendix, Activity 3, page 75.

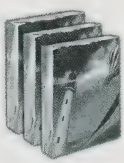


ACTIVITY 4

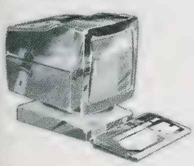
The Domain and Range of a Function

In Module 2 you worked with spreadsheet tables and reviewed how to use spreadsheet formulas to create these tables.

A knowledge of spreadsheets will help you understand more about mathematical relations. In particular, you will see how the idea of input values and output values can be applied to functions.



Turn to page 122 of your textbook and read Tutorial 3.3, "Graphing from Equations." **Note:** Stop reading at the bottom of page 122.



Next, work through the following steps, which explain how to use a computer spreadsheet to make the table and graph shown on the left-hand side of page 122 of the textbook.

Step 1: Enter the given input data in column A of a spreadsheet.

	A	B	C
1	−3		
2	−2		
3	−1		
4	0		
5	1		
6	2		
7	3		
8			

Step 2: Enter the formula **= A1+1** in cell B1.

Step 3: Click on cell B1 and drag down to cell B7. This action selects cells B1 through B7 in the B column. Next, click on **Edit** on the menu bar, and choose **Fill** and **Down**.

Your spreadsheet will now look like this.

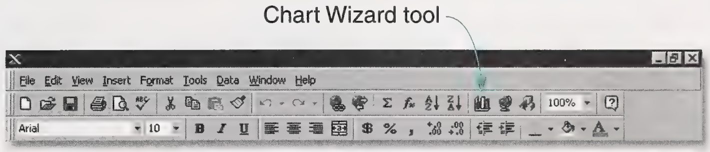
	A	B	C
1	−3	−2	
2	−2	−1	
3	−1	0	
4	0	1	
5	1	2	
6	2	3	
7	3	4	
8			

Step 4: Click on cell A1 and drag across and down to select the range of cells that you want to plot. **Note:** The selected cells will be highlighted.

	A	B	C
1	−3	−2	
2	−2	−1	
3	−1	0	
4	0	1	
5	1	2	
6	2	3	
7	3	4	
8			

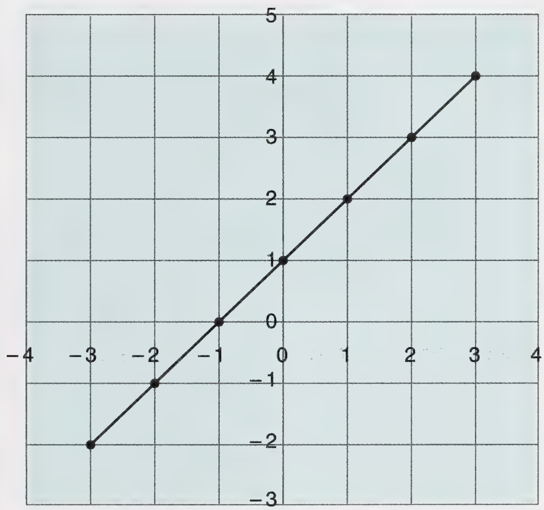
Step 5: Click on the Chart Wizard tool on the standard toolbar.

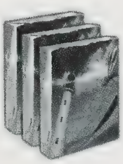
Note: The following illustration shows the Chart Wizard tool.



A Chart Wizard screen will appear. Choose **XY (Scatter)** under the Chart Type heading. Choose the scatterplot connected by smoothed lines, under the Chart Sub-type heading. Then, click on **Next** and **Finish**.

The graph of the data will look like this.





1. Using the spreadsheet formula = **A1^2**, make the table and graph shown on the right-hand side of page 122 of the textbook.

Compare your response with the suggested answer in the Appendix, Activity 4, pages 76–77.

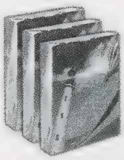
2. The spreadsheet formula **= A1+1** corresponds with the equation $y = x + 1$.

Write an equation with x and y that corresponds with the spreadsheet formula **= A1^2**.

Compare your response with the suggested answer in the Appendix, Activity 4, page 77.

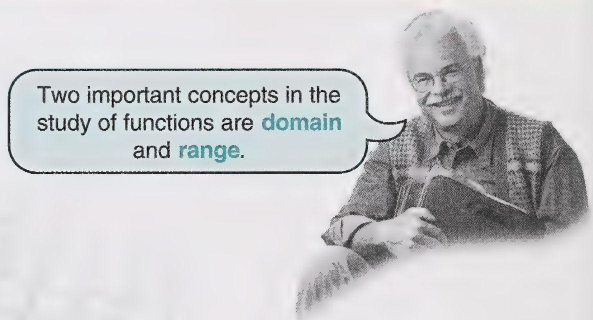
By working with spreadsheet formulas, you can see why the independent values (x-values) of a relation are **input values** and the dependent values (y-values) are **output values**.

The idea of input values and output values may help you better understand the concept of a function.



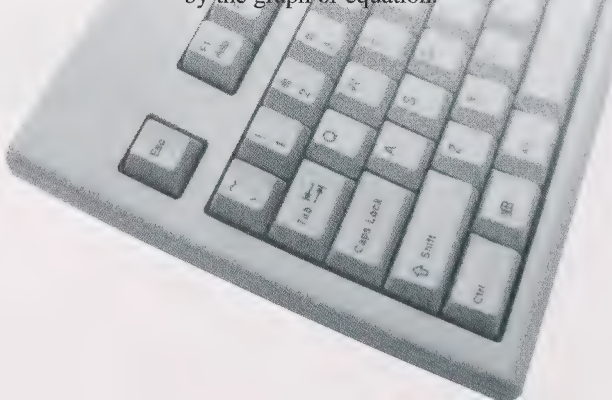
3. Turn to pages 126 and 127 of the textbook and answer exercises 1 and 6 of "Discussing the Ideas."

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 77–78.



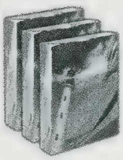
The domain of a function is the set of all possible x-values (valid input values) represented by the graph or equation.

The range of a function is the set of all possible y-values (valid ouput values) represented by the graph or equation.



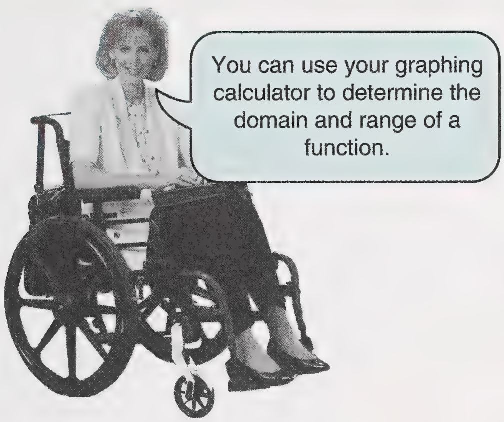
Although you probably haven't used the terms *domain* or *range* before, you have considered the possible x- and y-values every time you made a graph and asked yourself questions like these:

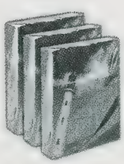
- Is it possible to have negative x-values?
- Is it possible to have rational x-values?
- Should the points on the graph be connected?
- Does the graph continue in the same pattern forever, or are there minimum and maximum values of x and y?



4. Turn to pages 143 and 144 of the textbook and answer exercises 3 and 4 of "Exercises: Checking Your Skills." **Note:** Make the graphs using a pencil-and-paper method. You will require graph paper.

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 78–79.



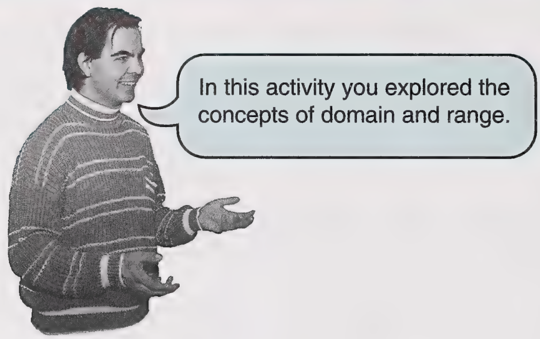


Turn to page 406 of the textbook and complete exercises 1, 2, and 3 of "Utility 10: Using the TI-83 to Find the Domain and Range of a Function."

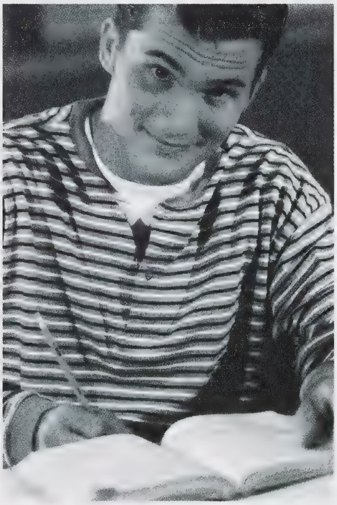
5. Turn to page 406 and answer exercises 4, 5, and 6 of "Utility 10: Using the TI-83 to Find the Domain and Range of a Function."

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 80–81.

LOOKING BACK



6. The TRACE and TABLE features of your graphing calculator can be used to determine the domain and range of a function.



What are other possible uses for the TRACE and TABLE features?

Write your answer in the journal section of your mathematics binder.

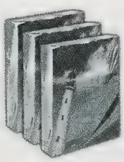
Compare your response with the suggested answer in the Appendix, Activity 4, page 81.

ACTIVITY 5

Representing Functions in Many Ways

People often have many names or labels that show how they are related to others. A mother may also be a daughter, a sister, a wife, a friend, and an employee.

Likewise, a function may be described in many ways. In this activity you will describe functions mathematically, visually, and symbolically. You will also use function notation.



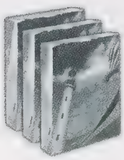
Turn to page 130 of the textbook and read the beginning of Tutorial 3.4, "Representing Functions in Many Ways." Then work through "Example 1: Interpreting a mapping diagram" and "Example 2: Deriving an equation from a table of values" on pages 130 and 131 of the textbook.

1. Turn to pages 132 to 134 of the textbook and answer exercises 1, 2, 3, and 5 of "Exercises: Checking Your Skills."

2. Turn to page 132 of the textbook and answer exercise 3 of "Discussing the Ideas."

Compare your responses with the suggested answers in the Appendix, Activity 5, pages 82–84.

FUNCTION NOTATION

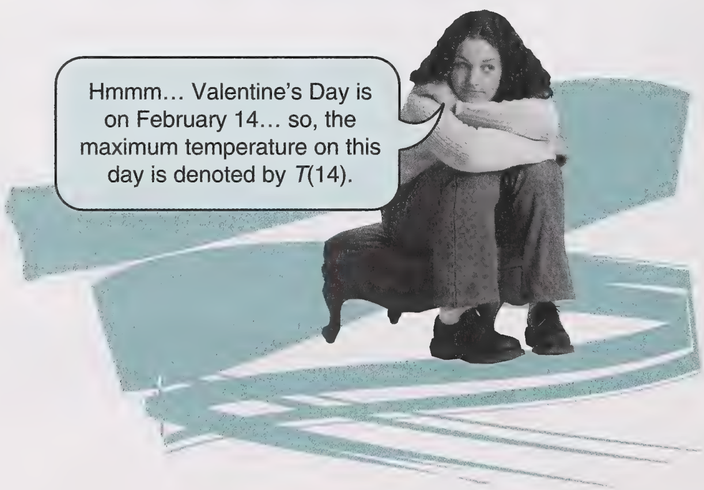


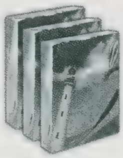
Calgary is known for its changeable weather. Turn to page 146 of the textbook and study the graph showing Calgary's daily maximum temperatures for February 1996. This graph is used to introduce you to another way of representing a function—using **function notation**.

Function notation is a "shorthand" way of writing a rule or relation.

For the function presented in your textbook, $T(d)$ stands for the maximum temperature for a day of February. For example, the notation $T(22)$ refers to the maximum temperature for February 22.

Based on this shorthand, what is the notation for the maximum temperature for Valentine's Day 1996?

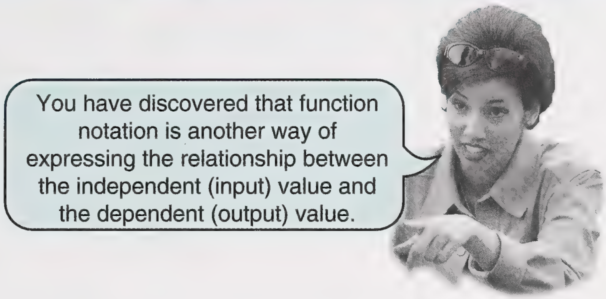




3. **a.** Turn to page 146 of your textbook and answer exercises 1 to 5 of Tutorial 3.6, “Function Notation.”

   b. Answer the matching activity at the bottom of page 146 of your textbook. That is, match exercises 1 to 5 at the top of the page with statements A to E on the bottom of the page.

Compare your responses with the suggested answers in the Appendix, Activity 5, page 84.



Work through the following example.

Example

A model rocket is fired upward. Its height, h, in metres above the ground is a function of time, t, in seconds. The function can be described by the following equation, written in function notation:

$$h(t) = 40t - 50t^2$$

What is the height of the rocket after 1 s? 2 s? 4 s? 6 s? 8 s?

Solution

If $t = 1$, then

$$\begin{aligned} h(t) &= 40t - 50t^2 \\ h(1) &= 40(1) - 5(1)^2 \\ &= 40 - 5(1) \\ &= 40 - 5 \\ &= 35 \end{aligned}$$

The height of the rocket after 1 s is 35 m.

If $t = 2$, then

$$\begin{aligned} h(t) &= 40t - 5t^2 \\ h(2) &= 40(2) - 5(2)^2 \\ &= 80 - 5(4) \\ &= 80 - 20 \\ &= 60 \end{aligned}$$

The height of the rocket after 2 s is 60 m.

If $t = 4$, then

$$h(t) = 40t - 5t^2$$
$$h(4) = 40(4) - 5(4)^2$$
$$= 160 - 5(16)$$
$$= 160 - 80$$
$$= 80$$

The height of the rocket after 4 s is 80 m.

If $t = 6$, then

$$h(t) = 40t - 5t^2$$
$$h(6) = 40(6) - 5(6)^2$$
$$= 240 - 5(36)$$
$$= 240 - 180$$
$$= 60$$

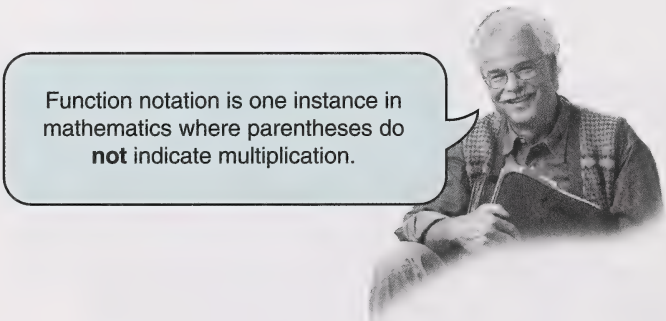
The height of the rocket after 6 s is 60 m.

If $t = 8$, then

$$h(t) = 40t - 5t^2$$
$$h(8) = 40(8) - 5(8)^2$$
$$= 320 - 5(64)$$
$$= 320 - 320$$
$$= 0$$

The height of the rocket after 8 s is 0 m.

The function notation $h(t)$ in the equation $h(t) = 40t - 5t^2$ represents the output value and can be replaced by y. The variable t in $40t - 5t^2$ represents the input value and can be replaced by x. Therefore, the equation $h(t) = 40t - 5t^2$ can be rewritten as $y = 40x - 5x^2$.



Turn to pages 147 and 148 of the textbook and work through "Example 1: Interpreting a cost equation."

4. **a.** Turn to page 150 of the textbook and answer exercise 2.b. of "Exercises: Checking Your Skills."

b. Use the given function $N(t) = 0.2t^2 + 4t + 50$ to determine the number of animals after 12 months.

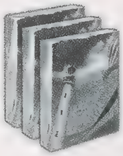
c. Rewrite the function $N(t) = 0.2t^2 + 4t + 50$ in terms of x and y.

d. Describe a reasonable domain and range for the function.

e. Define an appropriate WINDOW setting for graphing the function on a graphing calculator. Then graph the function.

f. Use the TABLE feature on your graphing calculator to determine the number of animals after 48 months.

Compare your responses with the suggested answers in the Appendix, Activity 5, pages 84–86.



5. a. Turn to page 150 of the textbook and answer exercises 1.a. and 1.d. of "Exercises: Checking Your Skills."

   b. Rewrite the function $C(F)=\frac{5}{9}(F-32)$ in terms of x and y.

   c. Describe a reasonable domain for the function.

   d. Define an appropriate WINDOW setting for graphing the function on a graphing calculator. Then graph the function.

   e. Use the TABLE feature of your graphing calculator to determine what 350°F is in degrees Celsius.

Compare your responses with the suggested answers in the Appendix, Activity 5, pages 86–87.

LOOKING BACK

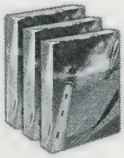
In this activity, you represented functions in many ways: in words, as a graph, as a table of values, as a mapping diagram, as a spreadsheet equation, as an equation in x and y, as a formula, and in function notation.

6. Find an example of each kind of representation in the textbook. Record the examples in the journal section of your mathematics binder.

Compare your response with the suggested answer in the Appendix, Activity 5, page 88.

Follow-up Activities

This module dealt with Chapter 3: Relations and Functions in the *Addison-Wesley Applied Mathematics 10 Source Book.*



Turn to the table on page 156 of the textbook and review the skills and concepts listed for each of the tutorials. Also, read the important results and formulas you discovered. The table summarizes what you should know and be able to do after participating in the learning activities of this module.

1. Turn to page 157 of the textbook and answer exercise 1 of Part A of "What Should I Be Able to Do?"

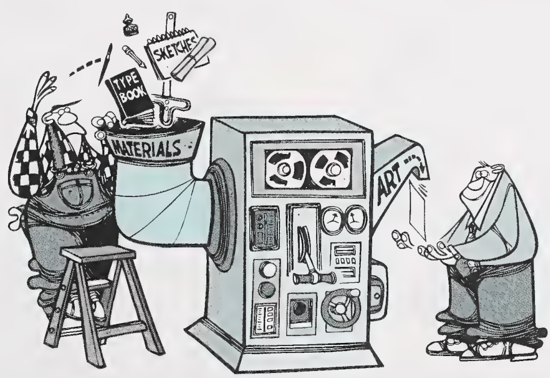
2. Turn to pages 158 and 159 of the textbook and answer exercises 2 to 6 of Part B of "What Should I Be Able to Do?"

Compare your responses with the suggested answers in the Appendix, Follow-up Activities, pages 88–92.

If you had difficulties understanding the skills and concepts in Module 3, Relations and Functions, it is recommended that you do the Extra Help. If you have a clear understanding of the skills and concepts in this module, it is recommended that you do the Enrichment. You may decide to do both.

EXTRA HELP

You should have a good grasp of function notation in order to deal with those relations called functions. It may help you to think of a function as an input-output machine.

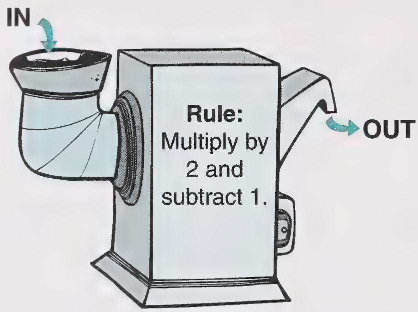


The machine may not be as fanciful as the one shown; but it does process numerical input values into output values according to any instructions you think the machine should follow.

Work through the following example.

Example

A function machine is set to multiply input values by 2 and then subtract 1.

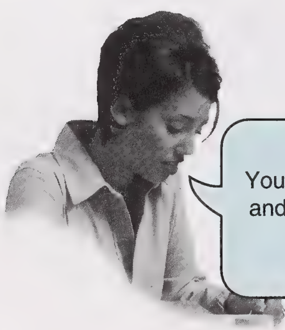


If the numbers 2, 0, and 5 are fed into the function machine, what will be the output values?

Solution

Input Values	Rule: Multiply by 2 and subtract 1.	Output Values
2	$2 \times 2 - 1$	3
0	$2 \times 0 - 1$	-1
5	$2 \times 5 - 1$	9

The set of all possible input values is called the domain. The set of all possible output values is called the range.



You may find it helpful to use the **shadow method** and the graph of a function to find the domain and range of a function.

1. Turn to pages 139 to 140 of the textbook and answer exercise 3 of "Discussing the Ideas." **Note:** Remember that if the graph of a function goes to the edge of the graphing calculator screen, it often means the graph continues in the same manner forever. In other words, only part of the graph is visible in the calculator display.

2. Turn to page 143 of textbook and answer exercises 1 and 2 of "Exercises: Checking Your Skills."

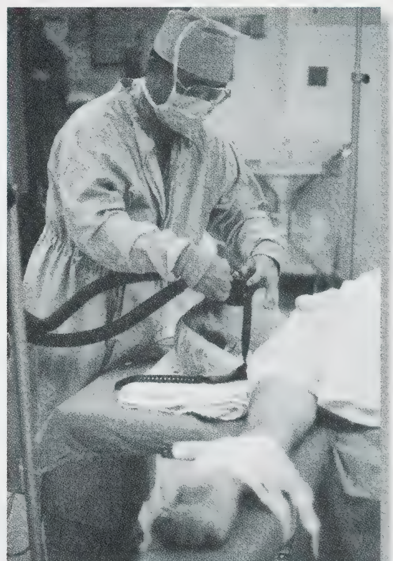
Compare your responses with the suggested answers in the Appendix, Follow-up Activities: Extra Help, pages 93–95.

ENRICHMENT

You can classify a function by its graph.

1. Turn to page 153 of the textbook and answer exercise 6.d. of "Exercises: Extending Your Thinking."

Compare your response with the suggested answer in the Appendix, Follow-up Activities: Enrichment, page 95.



View the video segment "Medical" on the companion CD.

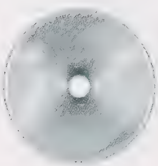
2. In the journal section of your mathematics binder, explain why a knowledge of functions is important for anesthesiologists.

Compare your response with the suggested answer in the Appendix, Follow-up Activities: Enrichment, page 96.

Completing the Project

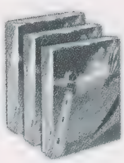
You should now have done most of the research for your Module 3 project, Epidemics.

To simulate a process is to produce a simple model that demonstrates the essential features of the process. Studying the model can make the fundamental nature of the process more understandable.



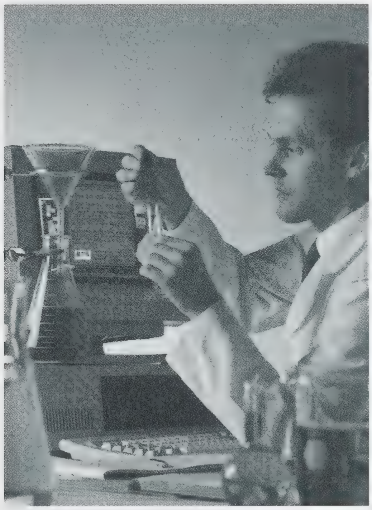
View "Epidemic" on the companion CD.

1. Turn to pages 136 and 137 of the textbook. Read the beginning of "Simulating the Spread of an Epidemic." Then answer exercises 1 to 4.



Compare your responses with the suggested answers in the Appendix, Module Project, pages 96–97.

2. The graph of the epidemic you made in textbook exercise 4 implies that an epidemic continues indefinitely. In reality, most epidemics last for only a short time. Give reasons to explain why an epidemic does not continue to grow indefinitely.



Compare your response with the suggested answer in the Appendix, Module Project, page 97.

Assessing a sample of student work can help you in the preparation of your own project.

3. Turn to page 161 of the textbook and answer exercises 8 and 9 of Part C of "What Should I Be Able to Do?"

Compare your responses with the suggested answers in the Appendix, Module Project, pages 97–98.

Module Project

Now that you have more insight into the module project, turn to your Project Booklet and complete the Module 3 project, Epidemics.

You may use your responses from the textbook exercises on pages 110, 136, 137, and 161 to help you complete the project (your responses should be in the project section of your mathematics binder).

Submit your completed Module 3 Project Booklet to your teacher.

Module Summary

In this module, you represented data using function models; represented functions with equations, graphs, and other forms; used a graphing tool to make graphs; used function notation to evaluate functions; and determined the domain and range of a function. You also became skilled at using function models to investigate phenomena in the real world.

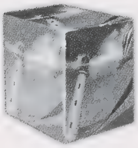
BETTMANN/CORBIS

Jonas Salk Displaying Polio Vaccine, 1955

The story of polio ended with a tremendous victory for humanity. Dr. Salk, a physician and epidemiologist, developed the first effective vaccine against polio. This vaccine became available in the 1950s. This first vaccine was injectable; an oral vaccine, developed by virologist Dr. Sabin, became available in 1960. By 1995, polio was considered eradicated from occurring naturally in the Western hemisphere. The World Health Organization is working towards having the disease eradicated globally early in the twenty-first century.

What facts and figures were you able to find for the epidemic you researched? Were you able to demonstrate, by using graphs and tables, how the disease could be spread?

Module Assignment



To demonstrate what you have learned in this module, complete the module assignment in the Assignment Booklet.

Submit your completed Module 3 Assignment Booklet to your teacher.

Applied Mathematics 10

APPENDIX

GLOSSARY

Dependent variable: the variable of a function whose value is determined by that of the independent variable of the function; the output or responding variable

Domain: the set of all possible x-values or independent variables of a relation; input values

Function: a relation in which there is only one value of the dependent variable for each value of the independent value; a relation in which there is only one output value for each input value

Function notation: a shorthand method of writing a procedure or rule that relates one number, quantity, and so on to another or others

Independent variable: the variable of a function whose value is specified first and determines the value of the dependent variable of the function; the input or manipulated variable

Range: the set of all possible y-values or dependent variables of a relation; output values

Scatterplot: a graph consisting of individual points whose coordinates represent values of two variables under investigation

SUGGESTED ANSWERS

Activity 1: Interpreting Graphs

1. a. The maximum solubility of sodium sulphate is about 49 g/100 mL. This value corresponds to the highest point on the graph.

b. The maximum solubility occurs at 40°C.

c. It is appropriate to connect the data points because the x-values and the y-values can be fractions.

2. a. The value of 2 nickels is 10¢. The value of 5 nickels is 25¢. The value of 8 nickels is 40¢.

b. It is not appropriate to connect the data points because the x-values and y-values cannot be fractions. For example, it is not possible to have $1\frac{1}{2}$ nickels.

3. a. The minimum fare charged is $1.50.

b. The fare charged for travelling 0.5 km is $1.50. The fare charged for travelling 1 km is $1.50. The fare charged for travelling 1.25 km is $3.00. The fare charged for travelling 2 km is $3.00.

c. It is appropriate to have open dots on this graph because two different fares cannot be charged for travelling the same distance. For example, the fare charged for travelling 2 km is $3.00, not $4.50.

4. **a.** The driver changes gears at about 40 km/h, 60 km/h, and 85 km/h. The flat parts of the curve show where there was no acceleration in speed. This would likely occur when the clutch was pushed in for a gear change.

   b. It is appropriate to connect the data points in this graph because the x-values and y-values can be fractions.

5. **Textbook exercises 2, 3, and 4 of "Exercises: Checking Your Skills," pp. 115 and 116**

   Responses to exercises 2, 3, and 4 may vary. Sample responses are given.

   2. **a.** A ball is thrown upwards. Once it reaches its highest point, if falls down again.

   b. Consider this as a price-versus-time graph for a long-distance telephone call. The charge goes up each minute, based on the per-minute charge. Time is rounded up to a whole number of minutes.

   c. This graph could represent the brightness of a flashlight bulb over time as the battery goes dead.

   d. This graph could represent the height of a campground water-pump handle, with respect to time, as someone fills a pail.

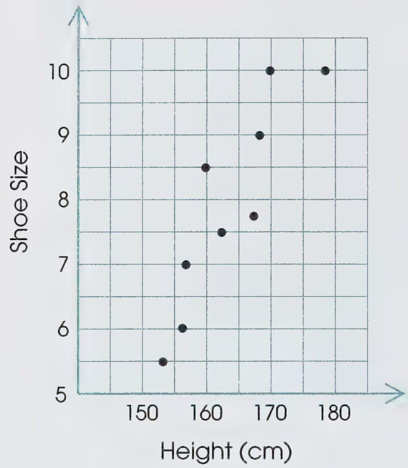
   3. Raoul slows down to cross an intersection. He then accelerates to his normal speed.

   4. Shakira decides to pick up her mail. Halfway to the mailbox, she takes time to do some stretches. Shakira then continues to the mailbox, where she takes a little longer stop in order to check and sort her mail. She then returns home, going at a constant speed.

6. The data points in a graph are connected if it is possible to have fractional x-values. The data points are not connected if the x-values are restricted to integers or specific fractional values.

Activity 2: Graphing from Tables of Data

1. Your scatterplot should look like the following.

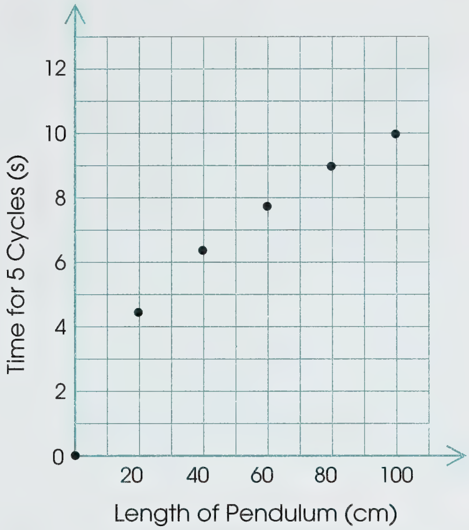


2. **Textbook exercises 1 to 6 of "Investigation 1: Length and Frequency of Pendulum," pp. 117 and 118**

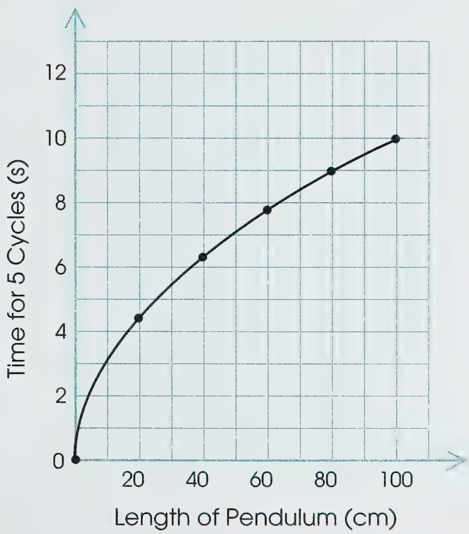
1. and 2. Your table should look like the following. The observed data may vary. A sample response is shown.

Length of Pendulum (cm)	Time for 5 Cycles (s)
100	10
80	9.0
60	7.8
40	6.3
20	4.5
0	0

3. Your scatterplot should look similar to the following scatterplot, which used the sample data from textbook exercises 1 and 2.

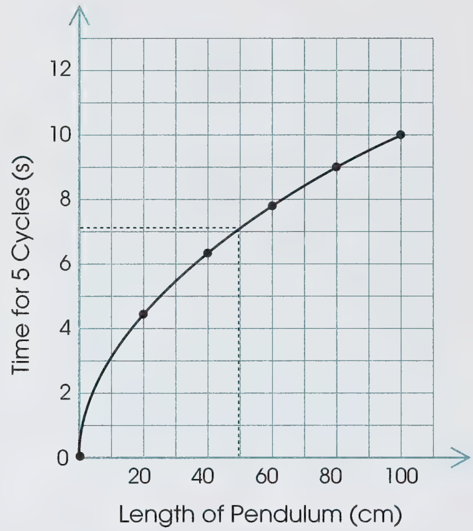


4. It would make sense to connect the points since a point (the pendulum length) can be adjusted to any length, including fractional lengths.



Activity 2 (continued)

5. According to the graph, it takes approximately 7.1 s for a 50-cm pendulum to swing 5 times.



$$\begin{aligned}\text{Period} &= \frac{\text{time}}{\text{number of swings}}\\ &= \frac{7.1}{5}\\ &\doteq 1.4\end{aligned}$$

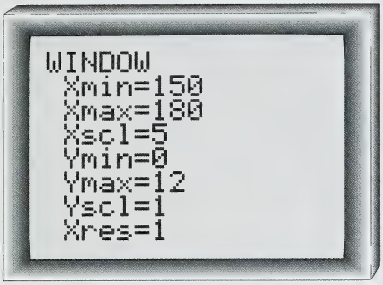
The 50-cm pendulum swings with a period of approximately 1.4 s.

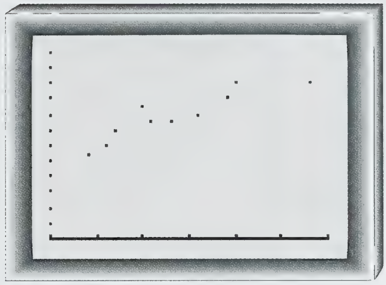
6. In textbook exercise 5, it was determined that a 50-cm pendulum swings 5 times in 7.1 s.

$$\begin{aligned}\text{Frequency} &= \frac{\text{number of cycles}}{\text{time}}\\ &= \frac{5}{7.1}\\ &\doteq 0.70\end{aligned}$$

The frequency of a 50-cm pendulum is approximately 0.70 cycles per second.

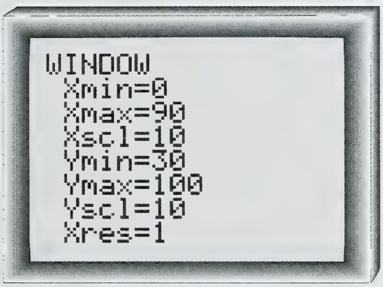
3. Graphs will vary slightly depending on the WINDOW settings chosen. Following is one possible window setting and the corresponding graph.

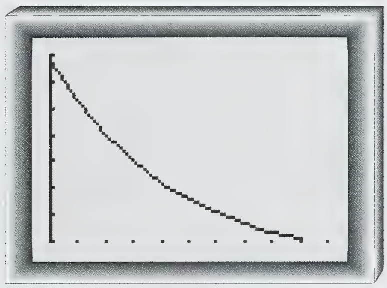




Note: Did you press 2nd [STAT PLOT] ENTER and select the first type of graph? This choice creates a graph with unconnected points.

4. **a.** Graphs will vary slightly depending on the WINDOW settings chosen. Following is one possible window setting and the corresponding graph.



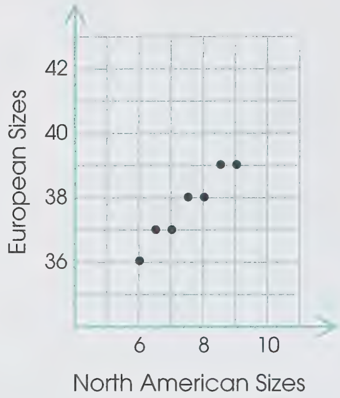


Note: Did you press 2nd [STAT PLOT] ENTER and select the second type of graph? This choice creates a graph with connected points.

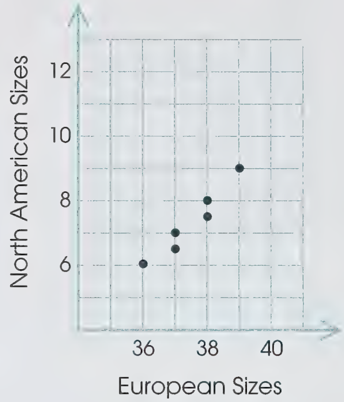
b. Yes the girls' theory was right. The liquid cooled faster initially.

Activity 2 (continued)

5. **a.** The scatterplot should look like the following.

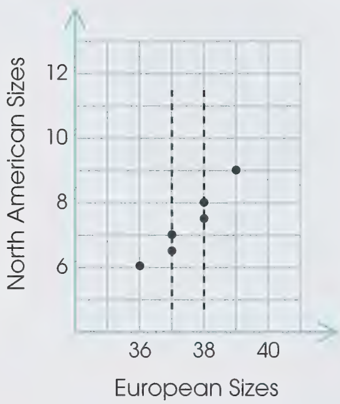


b. The scatterplot should look like the following.



6. The relation in question 5.a. is a function. A vertical line will not pass through two points.

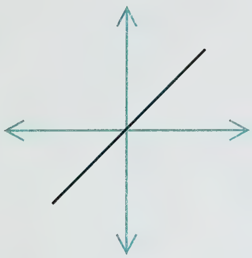
The relation in question 5.b. is not a function. A vertical line will connect two points, as shown in the following graph.



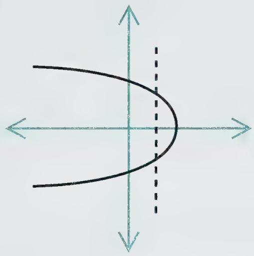
7. **a.** Yes, the graph is a function. A vertical line will not pass through two points.
   b. No, the graph is not a function. A vertical line will join two points.

8. A relation is a function when there is one and only one dependent value (y-value) for each independent value (x-value). A relation is not a function when there is more than one dependent value (y-value) for each independent value (x-value).

Examples will vary. Following are two possible examples using the vertical-line test.



This graph is a function. No two points can be joined by a vertical line.



This graph is not a function. Two points can be joined by a vertical line.

Activity 3: Graphing from Equations

1. Exercise based on data in "Example 1: Graphing Equations in *x* and *y*," p. 123

The calculations for the remainder of **a)** $y = 2x - 1$ are as follows:

If $x = -1$, then

$$\begin{aligned} y &= 2x - 1 \\ &= 2(-1) - 1 \\ &= -2 - 1 \\ &= -3 \end{aligned}$$

If $x = 0$, then

$$\begin{aligned} y &= 2x - 1 \\ &= 2(0) - 1 \\ &= 0 - 1 \\ &= -1 \end{aligned}$$

If $x = 1$, then

$$\begin{aligned} y &= 2x - 1 \\ &= 2(1) - 1 \\ &= 2 - 1 \\ &= 1 \end{aligned}$$

If $x = 2$, then

$$\begin{aligned} y &= 2x - 1 \\ &= 2(2) - 1 \\ &= 4 - 1 \\ &= 3 \end{aligned}$$

The calculations for the remainder of **b)** $y = 2^x$ are as follows:

If $x = 0$, then

$$\begin{aligned} y &= 2^x \\ &= 2^0 \\ &= 1 \end{aligned}$$

If $x = 1$, then

$$\begin{aligned} y &= 2^x \\ &= 2^1 \\ &= 2 \end{aligned}$$

If $x = 2$, then

$$\begin{aligned} y &= 2^x \\ &= 2^2 \\ &= 4 \end{aligned}$$

If $x = 3$, then

$$\begin{aligned} y &= 2^x \\ &= 2^3 \\ &= 8 \end{aligned}$$

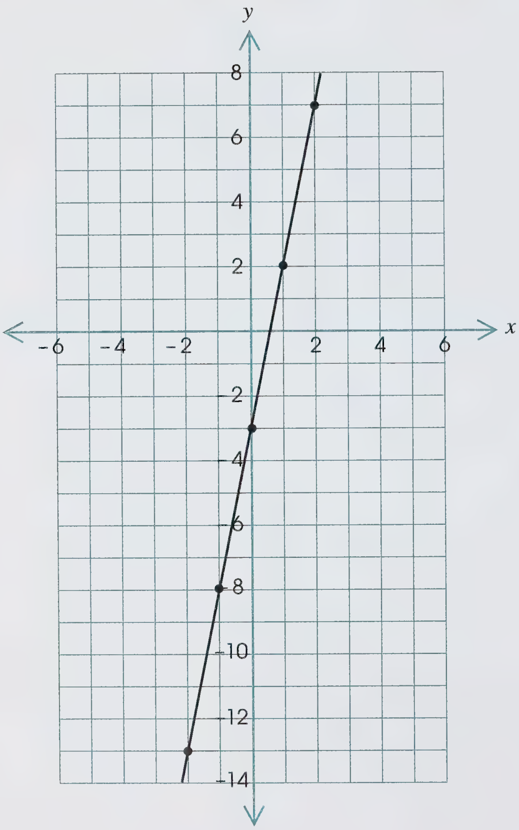
Activity 3 (continued)

2. **a.** $y = 5x - 3$

x	-2	-1	0	1	2
y	-13	-8	-3	2	7

When $x = 0$,

$$\begin{aligned} y &= 5x - 3 \\ &= 5(0) - 3 \\ &= 0 - 3 \\ &= -3 \end{aligned}$$

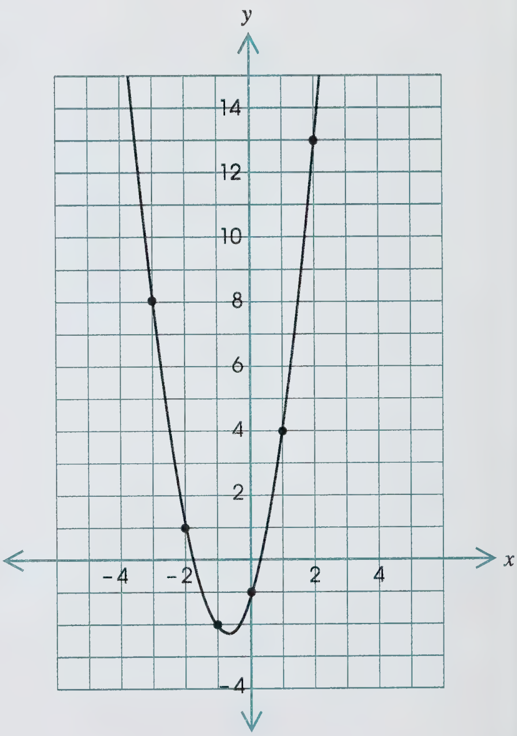


b. $y = 2x^2 + 3x - 1$

x	-3	-2	-1	0	1	2
y	8	1	-2	-1	4	13

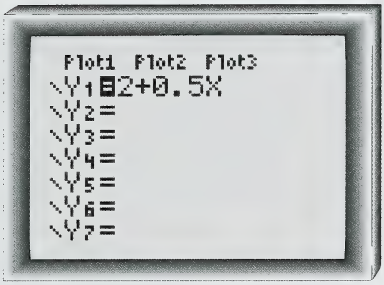
When $x = 0$,

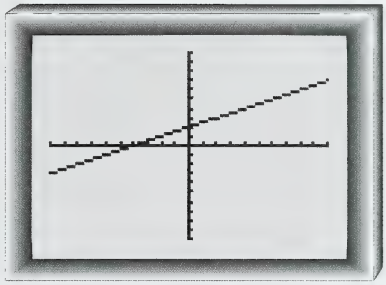
$$\begin{aligned} y &= 2x^2 + 3x - 1 \\ &= 2(0)^2 + 3(0) - 1 \\ &= 0 + 0 - 1 \\ &= -1 \end{aligned}$$



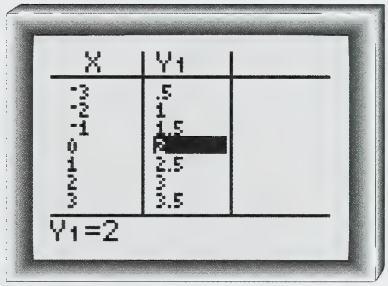
3. **Textbook exercises 1.c., 1.d., 1.e., and 1.h. of "Exercises: Checking Your Skills," p. 127**

1. c.



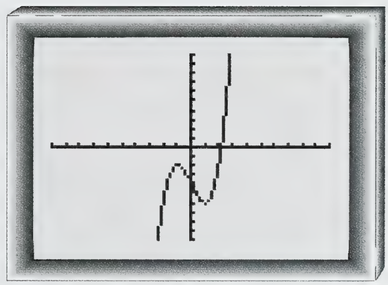


Use the table of values to determine the value of y when $x = 0$.

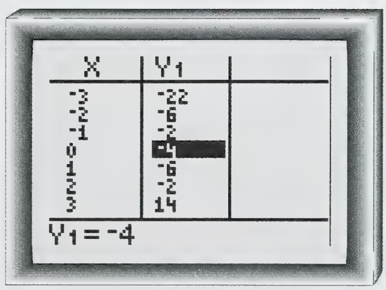


Therefore, $y = 2$ when $x = 0$.

d.

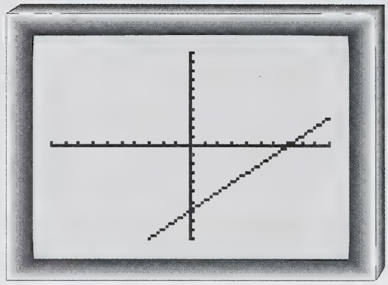


Use the table of values to determine the value of y when $x = 0$.

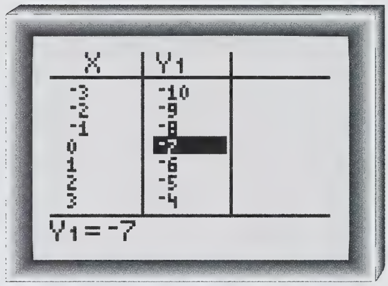


Therefore, $y = -4$ when $x = 0$.

e.

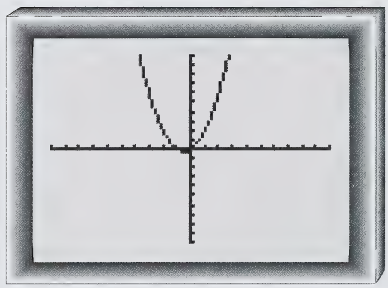


Use the table of values to determine the value of y when $x = 0$.

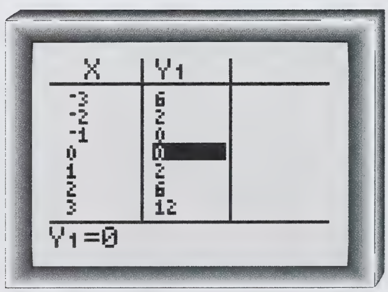


Therefore, $y = -7$ when $x = 0$.

h.



Use the table of values to determine the value of y when $x = 0$.



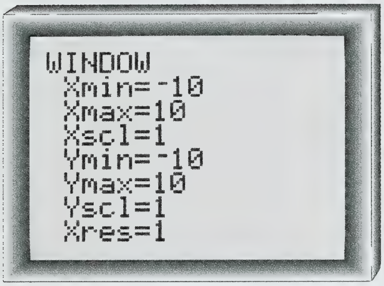
Therefore, $y = 0$ when $x = 0$.

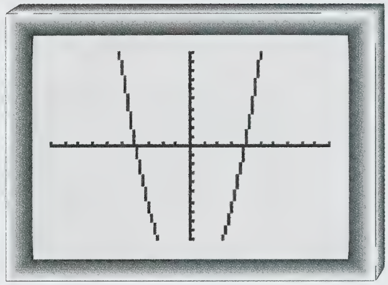
4. **a.** The graphs of $y = 2 + 0.5x$ and $y = x - 7$ are straight lines. Each of these equations has a degree of 1.

b. The graphs of $y = x^2 + x$ and $y = x^3 - 3x - 4$ are curves. Each of these equations has a degree greater than 1.

c. All the graphs are functions. For each graph, there is one and only one *y*-value for each *x*-value.

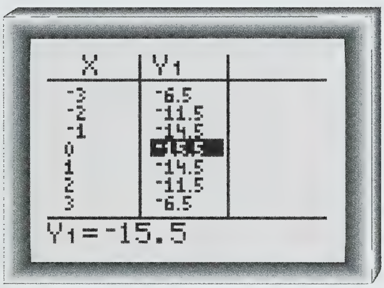
5. **a.** Using the default WINDOW settings, the graph of $y = x^2 - 15.5$ looks like the following.





Notice the bottom of the curve is cut off.

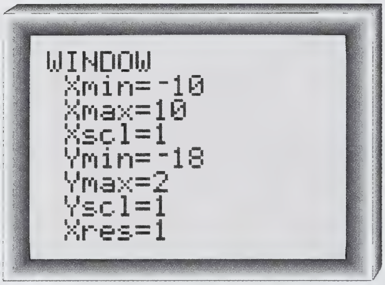
b. Use the TABLE view to estimate the lowest *y*-value.

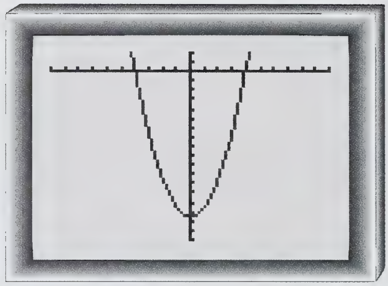


The lowest *y*-value is about -15.5.

Activity 3 (continued)

c. Adjust the WINDOW setting to include $y = -15.5$. Following is one acceptable WINDOW setting and the resulting graph.



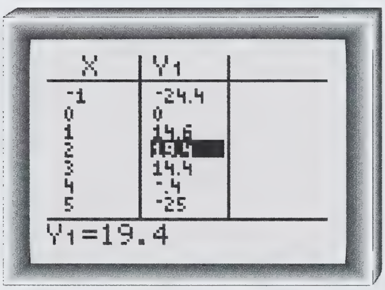


Notice that this is a clearer view of the curve.

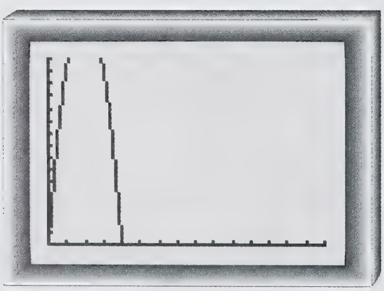
6. a. Time is the independent variable or x-value. Because time cannot be negative in this situation, a minimum x-value of 0 was chosen.

b. The speed is the dependent variable or y-value. Because speed cannot be negative in this situation, a minimum y-value of 0 was chosen.

c. From the TABLE view, it appears that the highest value of y is about 19.4.



d. No, the given WINDOW setting will not allow you to view the top of the curve. The WINDOW setting has $\text{Ymax} = 15$, but the highest value of y is about 19.4.

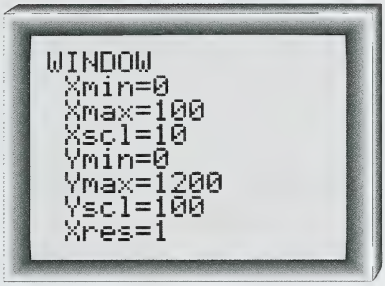


7. Textbook exercises 2 and 4 of Exercises: Checking Your Skills," pp. 127 and 128

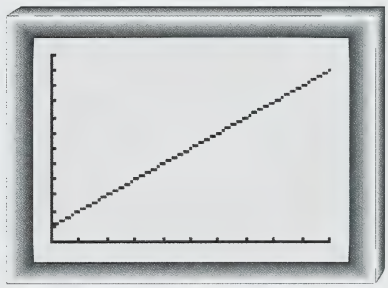
2. a. Step 1: Rewrite the formula $C = 100 + 10n$ using x- and y-variables.

$$y = 100 + 10x$$

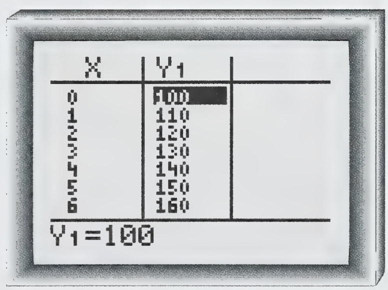
Step 2: Consider the restrictions and select appropriate WINDOW settings. WINDOW settings may vary. For this response, the following WINDOW settings were used.



Step 3: Graph the equation $y = 100 + 10x$.

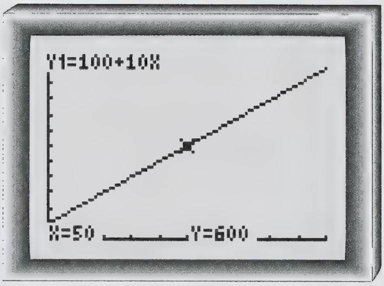


b. When no one attends a party, $x = 0$ (x represents the number of people attending the party). Therefore, the fixed cost is the value of y when $x = 0$. To determine the fixed cost, use the table of values on your graphing calculator (the TABLE view) and find the value of y when $x = 0$.



Therefore, the fixed cost is \$100.

c. Press TRACE . Find the y-value when $x = 50$.

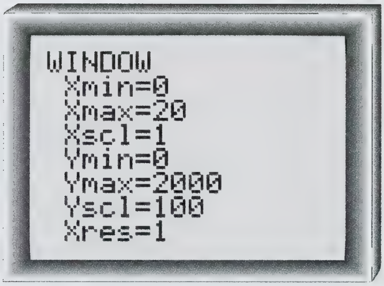


Using the TRACE feature, the cost of holding the party is \$600 if 50 people attend.

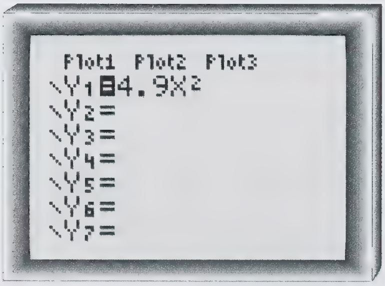
4. a. **Step 1:** Rewrite the formula $d = 4.9t^2$ using x- and y-variables. The y-value represents the distance travelled and the x-value represents the time.

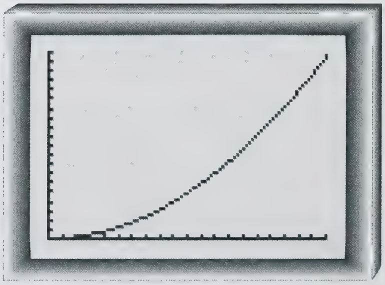
$$y = 4.9x^2$$

Step 2: Consider the restrictions and select appropriate WINDOW settings. WINDOW settings may vary. For this response, the given WINDOW settings were used.



Step 3: To graph the solution, enter the formula as $y = 4.9x^2$.

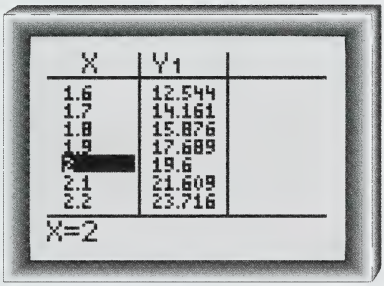


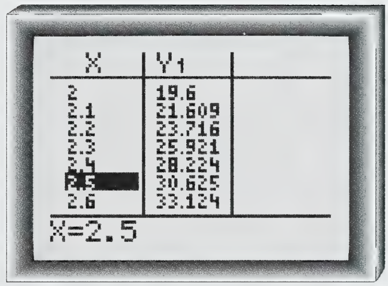


From the graph, you can see that the longer an object falls, the farther it falls. The distance the object travels increases from 0 m until it reaches the ground.

b. **Step 1:** Use the table of values to find the x-value that results in the y-value closest to 20. First, you will need to adjust the TABLE SETUP window.

Step 2: Find the x-values when $y \doteq 20$ and when $y \doteq 30$.





It would take an object approximately 2.0 s to fall about 20 m, and it would take approximately 2.5 s to fall about 30 m.

c. Not all objects fall at the same rate. Factors that change the air resistance that opposes the movement of an object affect the rate at which the object will fall. Other factors include the shape of the object, the density of the object, and the density of the air.

8. Answers will vary. Following is a sample response.

Advantages of using paper and pencil for graphing include the following:

- A graph may be drawn in a large size.
- Grid lines are visible.
- It is possible to refer to the graph at any time; it is a "hard" copy.

Advantages of using a calculator for graphing include the following:

- The calculator does calculations.
- The value of points is given as a digital readout rather than guessing.
- Values of points have greater precision.
- Graphing is quicker.

Activity 4: The Domain and Range of a Function

1. Exercise based on data in Tutorial 3.3, "Graphing from Equations," p. 122

Step 1: Enter the given input data in column A of a spreadsheet.

	A	B	C
1	−3		
2	−2		
3	−1		
4	0		
5	1		
6	2		
7	3		
8			

Step 2: Enter the formula **= A1^2** in cell B1.

Step 3: Click on cell B1 and drag down to cell B7. This action selects cells B1 through B7 in column B. Next, click on **Edit** on the menu bar and choose **Fill** and **Down**.

Your spreadsheet will now look like this.

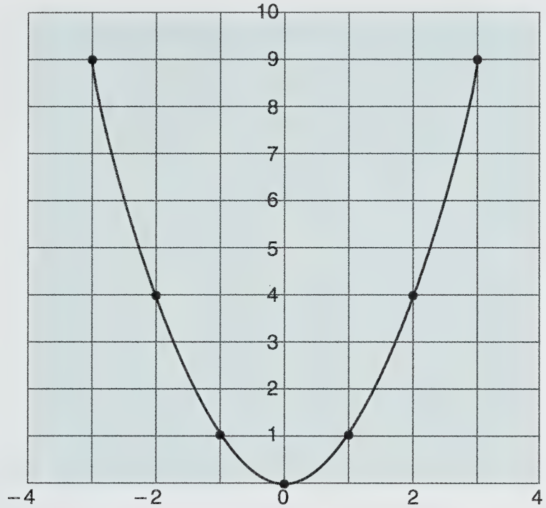
	A	B	C
1	−3	9	
2	−2	4	
3	−1	1	
4	0	0	
5	1	1	
6	2	4	
7	3	9	
8			

Step 4: Click on cell A1 and drag across and down to select the range of cells that you want to plot. **Note:** The selected cells will be highlighted.

	A	B	C
1	−3	9	
2	−2	4	
3	−1	1	
4	0	0	
5	1	1	
6	2	4	
7	3	9	
8			

Step 5: Click on the Chart Wizard tool on the toolbar. Then, click on the chart subtype that will produce a scatterplot connected by a line. Then, click on **Next** and **Finish**.

The graph of the data will look like this.



2. The formula is $y = x^2$.

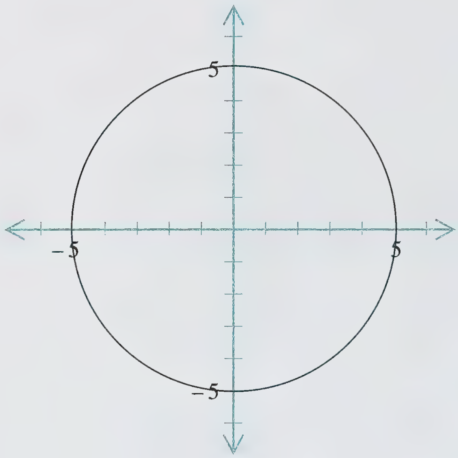
3. Textbook exercises 1 and 6 of "Discussing the Ideas," pp. 126 and 127

1. For the spreadsheet formula **= A1+1**, you would expect non-integer output values from non-integer input values. For example, if you input 1.5, the output is 2.5. Because rational values are permissible, the points in the graph can be connected.

$$\begin{aligned} y &= x+1 \\ &= (1.5)+1 \\ &= 2.5 \end{aligned} \qquad \begin{aligned} y &= x+1 \\ &= (2.25)+1 \\ &= 3.25 \end{aligned}$$

Activity 4 (continued)

6. The following is a graph of a cirle with centre (0, 0) and radius 5 units.



The graph is not a function because for some input values there are 2 output values; for example, for $x = 0$, $y = 5$ or -5.

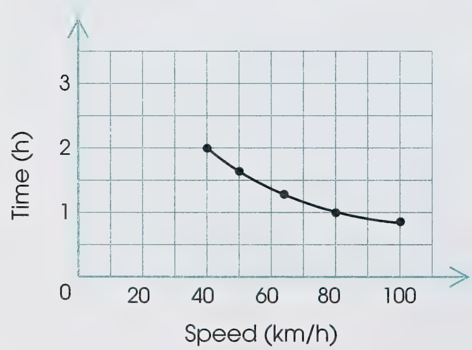
4. Textbook exercises 3 and 4 of "Exercises: Checking Your Skills," pp. 143 and 144

3. a.

Average Speed (km/h)	40	50	64	80	100
Time (h)	2	1.6	1.25	1	0.8

$$\text{time} = \frac{\text{distance}}{\text{average speed}}$$

b.



The points on the graph should be connected because the speed can be any value between 40 km/h and 100 km/h.

c. The possible values for speed are restricted. The driver drives no slower than 40 km/h and no faster than 100 km/h.

$\therefore\ 40 \leq \text{speed} \leq 100$

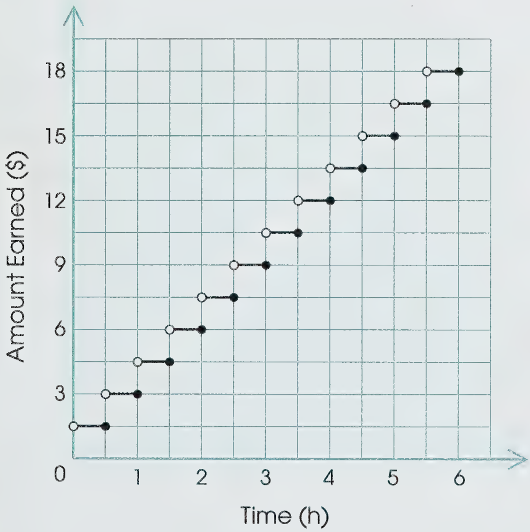
These values are the domain of the function.

d. The values for time depend on the speed. (See the table and/or graph.) In this situation, the time required is between 0.8 h and 2 h.

$\therefore\ 0.8 \leq \text{time} \leq 2$

These values are the range of the function.

4. a.

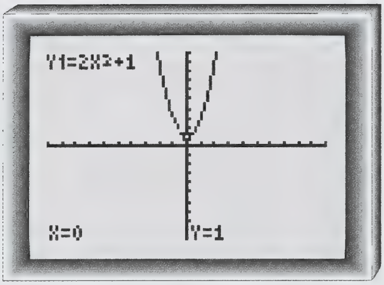


b. Because a single baby-sitting job can last up to 6 h, the domain of the graph is the set of all real numbers between 0 and 6.

Because Madison gets paid by the half hour and no baby-sitting job lasts longer than 6 h, the range of the graph is 1.5, 3.0, 4.5, 6.0, 7.5, 9.0, 10.5, 12.0, 13.5, 15.0, 16.5, and 18.0.

5. **Textbook exercises 4, 5, and 6 of Utility 10, "Using the TI-83 to Find the Domain and Range of a Function," p. 406**

4.



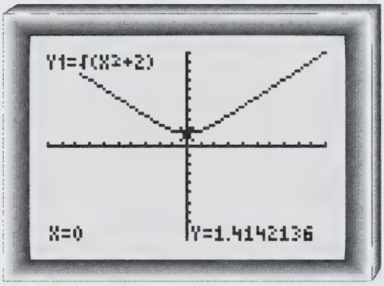
The lowest value of y is 1.

There is no highest value of y, no lowest value of x, and no highest value for x. The curve continues forever upward to the left and upward to the right.

The domain is the set of real numbers.

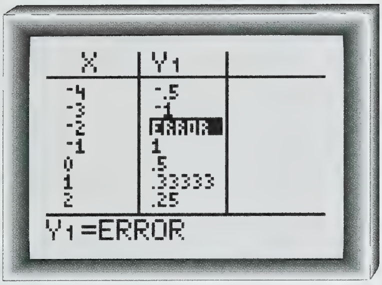
The range is the set of real numbers greater than or equal to 1.

5.



The domain is the set of real numbers.

The range is the set of real numbers greater than or equal to about 1.414 213 6.

6. 

Use the arrow keys to explore the table of values.

The value $x = -2$ is not permitted.

The domain is the set of all real numbers, except for $x = -2$.

Note: If -2 is substituted for x in the equation $y = \frac{1}{x+2}$, the result is an undefined value, as the following substitution shows.

If $x = -2$, then

$$\begin{aligned} y &= \frac{1}{x+2} \\ &= \frac{1}{-2+2} \\ &= \frac{1}{0} \\ &= \text{undefined} \end{aligned}$$

This explains why $x \neq -2$.

6. Answers will vary. A sample response is given.

The TABLE and TRACE features of your graphing calculator can be used to solve problems such as those on pages 127 and 128 of the textbook. Undefined values can be determined and values for specific values of x or y can also quickly be identified.

Activity 5: Representing Functions in Many Ways

1. Textbook exercises 1, 2, 3, and 5 of "Exercises: Checking Your Skills," pp. 132 to 134

1. a. i.

A	B
1	−1
2	0
3	1
4	2
5	3
6	4

ii.

H	K
−4	−8
−1	−2
0	0
3	6
5	10
9	18

iii.

J	$J+2$
−6	−4
−2	0
0	2
1	3
2	4
5	7

iv.

R	R^2
−3	9
−1	1
0	0
1	1
2	4
6	36

b. All the tables represent functions. For each table, any input value has just one output value.

c. **i.** Subtract 2 from the input value.
ii. Multiply the input value by 2.
iii. Add 2 to the input value.
iv. Square the input value.

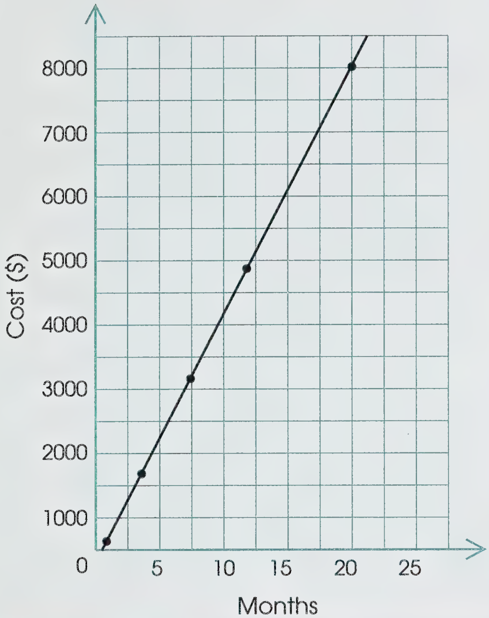
d. **i.** $B = A - 2$ **or** $y = x - 2$

ii. $K = 2H$ **or** $y = 2x$

2. a. You may have created a table of values that would display the value for each month. The following table displays the cost for every 4 months.

Months	Cost ($)
1	400
4	1600
8	3200
12	4800
16	6400
20	8000
24	9600

b.



c. $C = 400n$

3. a.

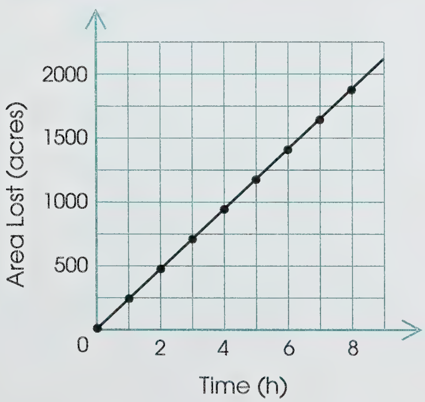
Time (in min)	0	1	10	20	29	30	40	50	59	60
Cost (in $)	0	2	2	2	2	4	4	4	4	6

b. The cost of parking is $2.00 for each 30 minutes of parking or part thereof.

5. a.

Time in hours	0	1	2	3	4	5	6	7	8
Total area lost (acres)	0	240	480	720	960	1200	1440	1680	1920

b.



c. $A = 240t$

Activity 5 (continued)

2. **Textbook exercise 3 of "Discussing the Ideas, p. 132**

    3. A mapping diagram represents a function if there is only one arrow starting from each input value. If there is even one input value having more than one arrow starting from it, the diagram represents a relation that is not a function.

3. a. **Textbook exercises 1 to 5 of Tutorial 3.6, "Function Notation," p. 146**

    1. The maximum temperature on February 3, 1996, was $-6°$ C.

    2. On February 1, 26, and 27 the temperature was $-18°$ C.

    3. On February 5 to 23 and on February 29 the maximum temperature was greater than 0°C.

    4. The temperature was rising from February 1 to 3, February 4 to 5, February 9 to 11, February 12 to 13, February 15 to 18, February 19 to 21, and February 27 to 29.

    5. On no day was the maximum daily temperature 0°C.

   b. **Textbook matching exercise of Tutorial 3.6, "Function Notation," p. 146**

    Statement A matches textbook exercise 4.
    Statement B matches textbook exercise 1.
    Statement C matches textbook exercise 3.
    Statement D matches textbook exercise 5.
    Statement E matches textbook exercise 2.

4. a. **Textbook exercise 2.b. of "Exercises: Checking Your Skills," p. 150**

    2. b. $N(t) = 0.2t^2 + 4t + 50$

$$\begin{aligned} N(0) &= 0.2(0)^2 + 4(0) + 50 \\ &= 0.2(0) + 0 + 50 \\ &= 0 + 0 + 50 \\ &= 50 \end{aligned}$$

    The number of animals when the scientist began was 50.

$$
\begin{aligned}
N(t) &= 0.2t^2 + 4t + 50 \\
N(1) &= 0.2(1)^2 + 4(1) + 50 \\
&= 0.2(1) + 4 + 50 \\
&= 0.2 + 4 + 50 \\
&= 54.2
\end{aligned}
$$

The number of animals after 1 month was about 54.

$$
\begin{aligned}
N(t) &= 0.2t^2 + 4t + 50 \\
N(-1) &= 0.2(-1)^2 + 4(-1) + 50 \\
&= 0.2(1) - 4 + 50 \\
&= 0.2 - 4 + 50 \\
&= 46.2
\end{aligned}
$$

The number of animals 1 month before the scientist began the project was about 46.

b.
$$
\begin{aligned}
N(t) &= 0.2t^2 + 4t + 50 \\
&= 0.2(12)^2 + 4(12) + 50 \\
&= 0.2(144) + 48 + 50 \\
&= 28.8 + 48 + 50 \\
&= 126.8
\end{aligned}
$$

There were about 127 animals after 12 months.

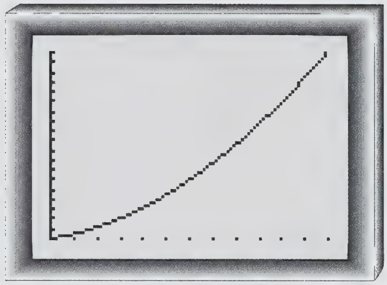
c. The function $N(t) = 0.2t^2 + 4t + 50$ can be rewritten as $y = 0.2x^2 + 4x + 50$.

d. The time is the independent variable. The scientist may have been studying the animals for several years. A reasonable length for the study is probably 5 years (or $5 \times 12 = 60$ months). Therefore, it makes sense for the domain to be the values from 0 to 60.

The number of animals is the dependent variable. There were 50 animals at the beginning of the project and 127 animals after 12 months. A reasonable range might be from 50 to 1000.

e. An appropriate WINDOW setting and the corresponding graph are shown here.

```
WINDOW
 Xmin=0
 Xmax=60
 Xscl=5
 Ymin=50
 Ymax=1000
 Yscl=50
 Xres=1
```



f. Use the TABLE view to find the number of animals (y) after 48 months (x).

X	Y_1
48	702.8
49	726.2
50	750
51	774.2
52	798.8
53	823.8
54	849.2

$Y_1=702.8$

There were about 703 animals after 48 months.

5. a. Textbook exercises 1.a. and 1.d. of "Exercises: Checking Your Skills," p. 150

1. a. The equation $C(50)=10$ means that a temperature of $50°\text{F}=10°\text{C}$.

$$C(F)=\frac{5}{9}(F-32)$$

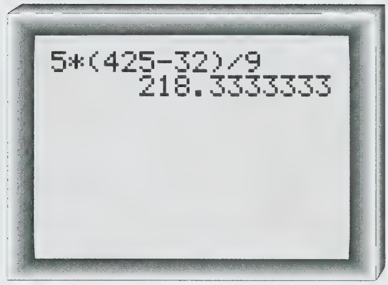
$$C(50)=\frac{5}{9}(50-32)$$

$$=\frac{5}{\cancel{9}_{1}}\times\cancel{18}^{2}$$

$$=10$$

d. $C(F)=\frac{5}{9}(F-32)$

$$C(425)=\frac{5}{9}(425-32)$$
$$=\frac{5}{9}(393)$$
$$=218$$

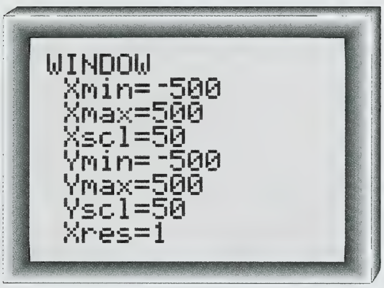


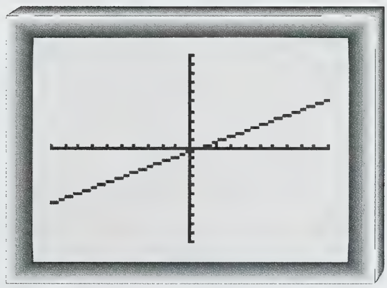
The temperature of 425°F is equivalent to 218°C.

b. The function $C(F)=\frac{5}{9}(F-32)$ can be rewritten as $y=\frac{5}{9}(x-32)$.

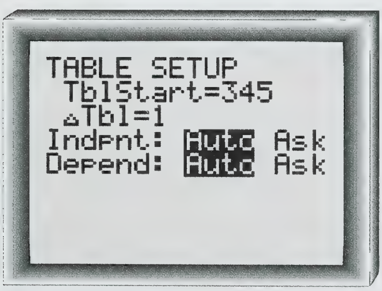
c. A reasonable domain would be the values from -500 to 500.

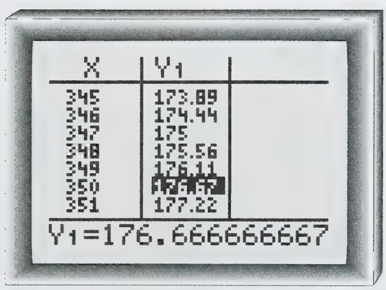
d. An appropriate WINDOW setting and the corresponding graph are given here.





e. In this sample response, the table is started with 345. The table increment is kept at 1. **Note:** See Utility 9 if you need extra help in changing the table setup. Press (2nd) [TBLSET]. Change the x-value to the number you wish the table to start with. You may also wish to change the increment value. Changing the table setup speeds up the task of locating some values. You could use the arrow keys to scroll through the table of values.





A temperature of 350°F is equivalent to about 177°C.

Activity 5 (continued)

6. Answers will vary. A sample response is given.

- function in words: p. 134, textbook exercise 4, “words” column
- function as a graph: p. 114, Graph 1
- function as a table of values: p. 131, Example 1, solution b)
- function as a mapping diagram: p. 131, Example 1, i)
- function as a spreadsheet equation: p. 122, Tutorial 3.3
- function as an equation in x and y: p. 123, Example 1
- function as a formula: p. 125, Example 2
- function in function notation: p. 149, exercise 1

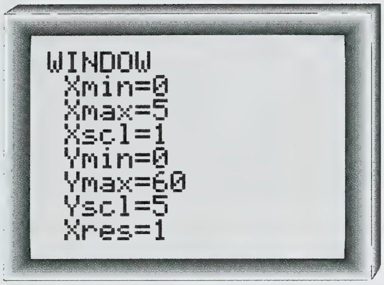
Follow-up Activities

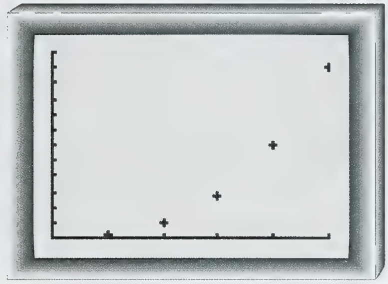
1. Textbook exercise 1 of Part A of “What Should I Be Able To Do?” p. 157

1. a. A square-based pyramid with 2 layers uses 5 marbles. It is reasonable to consider 1 as pyramidal. The number 1 corresponds to the top layer of every pyramid. The number 1 is also a perfect square, as is the number of balls in each layer of square-based pyramids.

Layers	Pyramidal Numbers
1	1
2	5
3	14
4	30
5	55

b. Define appropriate WINDOW settings and graph the table of values.





Note: The dots should not be connected because the function only makes sense for natural numbers 1, 2, 3, …, which correspond to layer numbers.

c. Square the new layer number. Add this square to the previous pyramidal numbers.

d. The domain is the natural numbers. The range is the set of all pyramidal numbers.

e. The relation is a function because for each layer (input number) there is only one pyramidal number (output number).

f. Use a table to show the relationship between the number of layers and the corresponding pyramidal number, expressed as a sum of squares.

Layer	Pyramidal Number as a Sum of Squares
(1)	(1^2)
2	2^2+1^2
3	$3^2+2^2+1^2$
4	$4^2+3^2+2^2+1^2$
5	$5^2+4^2+3^2+2^2+1^2$
6	$6^2+5^2+4^2+3^2+2^2+1^2$

If P represents the value of the 6th pyramidal number, the expression would be $P=6^2+5^2+4^2+3^2+2^2+1^2$.

Follow-up Activities (continued)

2. Textbook exercises 2 to 6 of Part B of "What Should I Be Able to Do?" pp. 158 to 159

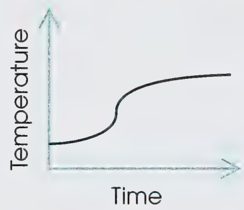
2. Answers will vary. Sample responses are given.

a. The graph could describe a situation where a person is bicycling up a hill and the speed decreases. Then the speed increases as the person goes downhill.

b. The graph could describe the amount of water in a bathtub over a specified time period. The amount of water increases at a steady rate as the tub is filled, remains at a certain level during the bath, and decreases as the water is drained from the tub.

3. a. Time is the independent variable. Temperature is the dependent variable.

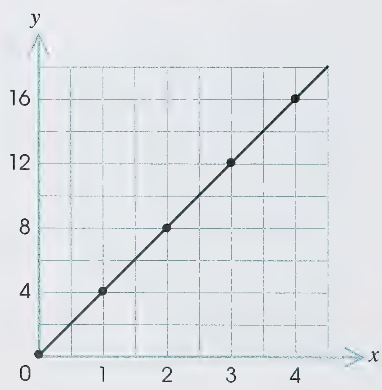
b.



4. The table does not represent a function. The input value of 9 has more than 1 output value.

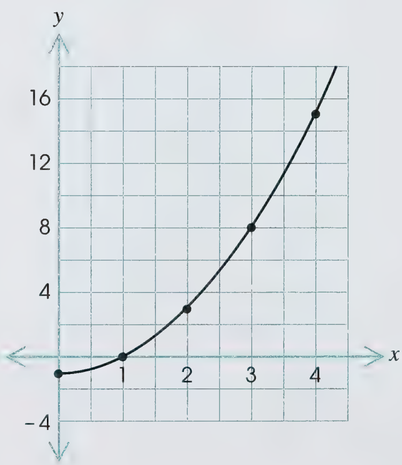
5. a. and b.

Graph 1



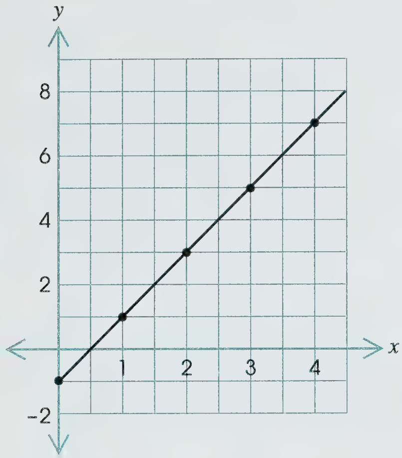
The rule is $y = 4x$.

Graph 2



The rule is $y = x^2 - 1$.

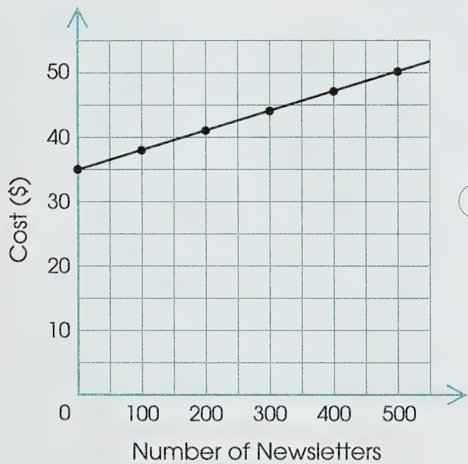
Graph 3



The rule is $y = 2x - 1$.

6. a.

Number of Newsletters	0	100	200	300	400	500
Cost ($)	35	38	41	44	47	50



The function $C(n) = 35 + 0.03n$, in terms of x and y, is $y = 35 + 0.03x$.

b. The domain (the number of newsletters) is all whole numbers greater than or equal to 0. The range (the cost in dollars) is all the rational numbers greater than or equal to 35.

c. **Method 1: Using the Formula**

$$C(n) = 35 + 0.03n$$
$$C(1000) = 35 + 0.03(1000)$$
$$= 35 + 30$$
$$= 65$$

The cost of 1000 newsletters is $65.

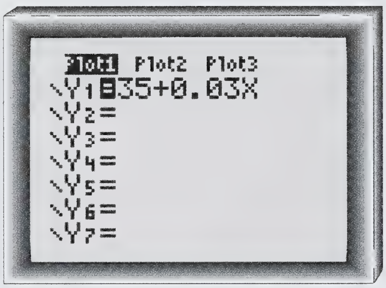
Method 2: Using a Calculator

Step 1: Rewrite the formula using *x*- and *y*-values. The formula would be as follows:

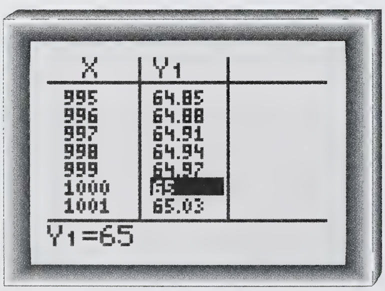
$$y = 35 + 0.03x$$

In this formula, *y* represents the cost of the newsletters; *x* represents the number of newsletters.

Step 2: Enter the formula in your calculator.



Step 3: Use the TABLE view to detemine the value when $x = 1000$.



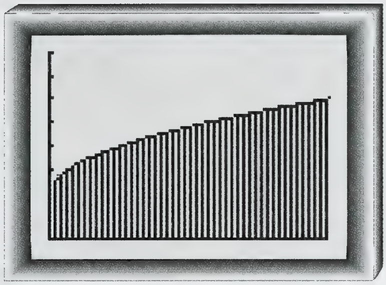
The cost of 1000 newsletters is $65.

d. The cost of printing 250 newsletters is represented by $C(250)$.

Extra Help

1. **Textbook exercise 3 of "Discussing the Ideas," pp. 139 to 140**

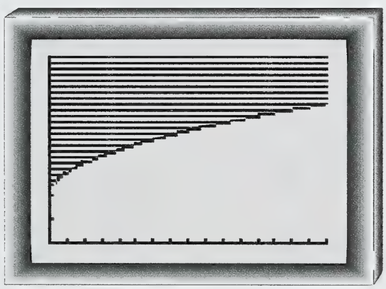
 3. **a.** The values of x that are in the shadow are all the x-values greater than or equal to 0. (Remember that the screen only shows part of the graph. The graph continues upwards to the right.)



The shadow will continue to the right.

The domain is the set of all x-values greater than or equal to 0. Since each x-value casts a shadow on the x-axis, the domain of the function is given by all the values of x that fall in the shadow.

b. The values of y that are in the shadow are all the y-values greater than or equal to 2. (Remember that the screen only shows part of the graph. The graph continues upwards to the right.)



The shadow will continue upwards.

The range is the set of all the y-values greater than or equal to 2. Since each y-value casts a shadow on the y-axis, the range of the function is given by all the values of y that fall in the shadow.

c. The domain is all the real numbers greater than or equal to 0. The domain can be written as the following:

$$x \geq 0, x \in R$$

The range is the real numbers greater than or equal to 2. The range can be written as the following:

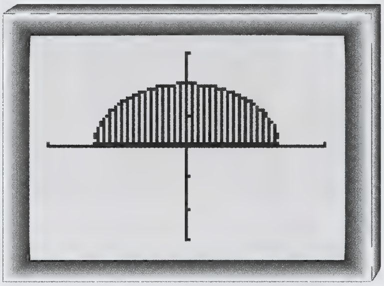
$$y \geq 2, y \in R$$

Follow-up Activities (continued)

2. **Textbook exercises 1 and 2 of "Exercises: Checking Your Skills," p. 143**

   1. **Step 1:** Visualize a light shining from directly above the graph. The graph throws a shadow on a portion of the *x*-axis.

      The shadow will fall on all the *x*-axis from -4 to 4 inclusive.

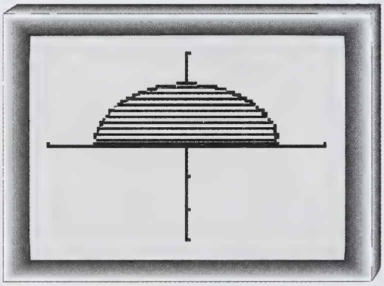
      

      The domain is the set of real numbers between -4 and 4 inclusive. The domain can be written as the following:

      $-4 \leq x \leq 4, x \in R$

      Step 2: Visualize a light shining from the right and left sides of the graph. The graph throws a shadow on a part of the *y*-axis.

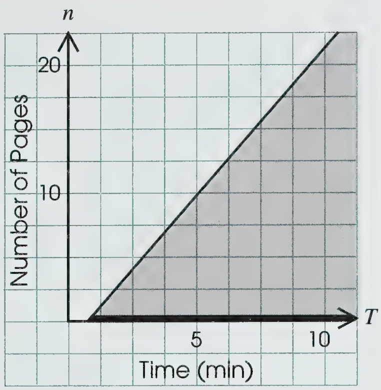
      The shadow will fall on all the *y*-axis from 0 to 2 inclusive.

      The range is the set of real numbers from 0 to 2 inclusive. The range can be written as the following:

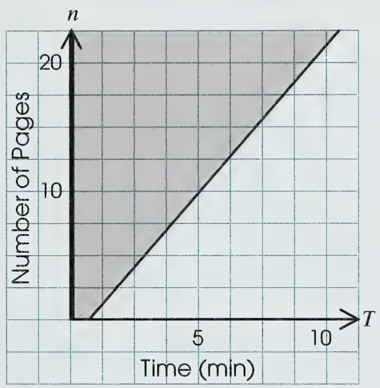
      $0 \leq y \leq 2, y \in R$

2. a.



The shadow will continue to the right.

The domain is all the real numbers greater than 0.5.



The shadow will continue upwards.

The range is all the real numbers greater than 1.

b. When $n = 0$, $T = 0.5$. This minimum represents the time the printer requires to process any request.

Enrichment

1. Textbook exercise 6.d. of "Exercises: Extending Your Thinking," p. 153

6. d. Answers will vary. Following is a sample response.

The graphs of the equations in exercises 1.a., c., e., and g. of "Exercises: Checking Your Skills" on page 127 are linear functions.

The graphs of the equations in exercises 1.b. and 1.f. of "Exercises: Checking Your Skills" on page 127 are quadratic functions.

The graph of the equation in exercise 1.d. of "Exercises: Checking Your Skills" on page 127 is a cubic function.

The graph in Example 1, equation b), on page 123 is a power function.

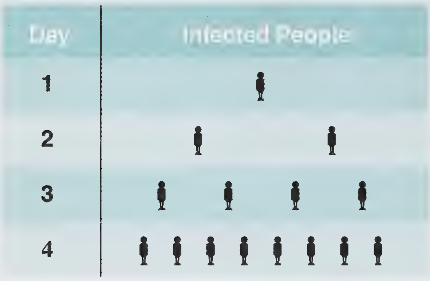
Follow-up Activities (continued)

2. The relationship between the amount of sodium pentathol administered and the time is not a simple linear function. The anesthesiologist must understand functions so that the patient is given the right amount of drug.

Module Project: Epidemics

Completing the Project

1. **Textbook exercises 1 to 4 from "Simulating the Spread of an Epidemic," pp. 136 to 137**

   1. 

   2. There are 8 people infected on the fourth day. The total number of infected people is 15.

      $1 + 2 + 4 + 8 = 15$

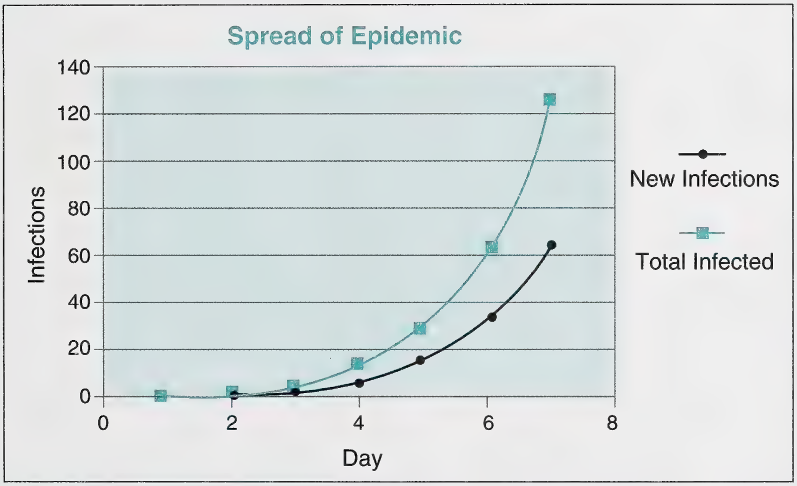
   3. Your spreadsheet should look like the following.

	A	B	C
1	Day	New Infections	Total Infected
2	1	1	1
3	2	2	3
4	3	4	7
5	4	8	15
6	5	16	31
7	6	32	63
8	7	64	127

The following shows the formula view of the spreadsheet.

	A	B	C
1	Day	New Infections	Total Infected
2	1	1	1
3	2	=B2*2	=C2+B3
4	3	=B3*2	=C3+B4
5	4	=B4*2	=C4+B5
6	5	=B5*2	=C5+B6
7	6	=B6*2	=C6+B7
8	7	=B7*2	=C7+B8

4. Your graph should be similar to the following. **Note:** Select cells A1 to C8 and proceed to the Chart Wizard tool. Use the Chart Options to finish the graph.



The graph shows that the number of new infections and the total number infected starts slowly and increases at a rapid rate.

2. An epidemic does not continue to grow indefinitely because there are other competing forces. Medical personnel usually intervene to administer to the ill people and the infected individuals may be quarantined to slow the spread of the infection. Some people have immunity against the disease and don't get sick. For some illnesses, people are only contagious during a certain stage of the disease. Geography can also play a part in controlling the spread of a disease.

3. **Textbook exercises 8 and 9 of Part C of "What Should I Be Able to Do?" pp. 161 and 162**

8. Answers will vary. You may have observed the following:

- The chart showing numbers is accurate and utilizes a spreadsheet well.
- There should not be a row for day 0; the epidemic starts on day 1.

- The numercial results are correct; the formulas are appropriate.
- The graph is not easy to read.
  - There is one extra curve (for the day). This is due to selecting "Line" for the chart type rather than "Scatterplot."
  - The x-axis is difficult to read, having values displayed between tick marks. Selecting "Scatterplot" for the chart type would fix this.
  - The x-axis should be labelled "Day."
  - The y-axis should be labelled "Infections."
  - The graph should have a title.

9. For the most part, the topic is treated knowledgeably and respectfully. However, some spelling and grammatical errors detract from the overall positive impression left by the brochure.

- The facts presented seen plausible. The flow, as indicated by the subtitles, is logical.
- The mathematical patterning behind the numbers infected is not adequately communicated. However, the magnitude of the threat of an unchecked ebola epidemic is clearly conveyed. This magnitude suggests that an ebola epidemic would grow exponentially. The question of how the disease spreads is not answered.
- The graphics and layout of the text are effective. The graphics, representing death and biohazard, attract the reader to the article. The picture of the virus makes its structure more easily understood. Perhaps some indication of the scale used to enlarge the drawing could have been given.
- Parts of the text appear to have been copied without acknowledgement as to the source. Terms such as *bacilliform*, *nucleocapsid*, and *cross-straited helical capsid* should be defined or simplified. They are too complex for the project and actually detract from the content.

The brochure is visually attractive and informative. However, a greater emphasis should have been placed on describing the mathematical pattern of the spread of the disease and giving statistics relating to the outbreak of the disease.

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